Analyzing Coverage Probability of Reconfigurable Intelligence Surface-aided NOMA

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Abstract—Along with the explosive growth of wireless communication network users who require large frequency bands and low latency, it is a challenge to create a new wireless communication network beyond 5G. This is because installing a massive 5G network requires a large investment by network providers. For this reason, the authors propose an alternative beyond 5G that has better quality than 5G and a relatively lower investment value than 5G networks. This study aims to analyze the downlink of the cooperative non-orthogonal multiple access (NOMA) network, which is usually used in 5G, combined with the use of a reconfigurable intelligence surface (RIS) antenna with decode and forward relay mechanisms. RIS is processed with a limited number of objects utilizing Rayleigh fading channels. The scenario is created by a user who relays without a direct link for users near the base station and with a direct link for users far from the base station. Under the Nakagami-m fading channel, the authors carefully evaluated the probability of loss for various users as a function of perfect channel statistical information (p-CSI) utilizing simply a single input-output (SISO) system with a finite number of RIS elements. As a key success metric, the efficiency of the proposed RIS-assisted NOMA transmission mechanism is evaluated through numerical data on the outage probability for each user. The modeling outcomes demonstrate that the RIS-aided NOMA network outperforms the traditional NOMA network.

Keywords—Non-orthogonal multiple access (NOMA); outage probability (OP); p-CSI.

I. INTRODUCTION

Reconfigurable intelligent surfaces (RIS) are viewed as cutting-edge technology for the beyond fifth generation (B5G) communication system due to their potential to produce considerable increases in communication coverage, throughput, and energy effectiveness [1]-[6]. According to Liaskos et al. [7], due to the vast number of inexpensive reflecting devices that make up RIS, it is possible to cleverly rearrange the reflected signal propagation to meet specific communication objectives by changing the phase shifts of every reflecting unit [8], [9]. RIS is a planar meta-surface combined with a number of passive parts that can dynamically vary their reflections for various applications, including enhancing signal strength and reducing interference [10], [11]. In comparison to traditional techniques like active relaying and beam shaping. RIS reflects signals in a full-duplex and noise-free manner and significantly reduces energy consumption and hardware or deployment costs by utilizing only lightweight passive elements. [12], [13]. By altering the phase shifts of its passive components, RIS can artificially boost combined channel strengths and enlarge channel strengths using reflected electromagnetic waves [14]. Future wireless communication systems may benefit from NOMA, also known as non-orthogonal multiple access, which can assist several users inside a single resource part [15]-[17]. In conventional wireless networks without RIS, NOMA has concerned much attention and has shown to be an improvement over orthogonal multiple access (OMA) [18], [19]. NOMA can boost spectrum effectiveness while complementary user fairness and boosting network connections. The user of the robust channel employs a
successive interference cancellation (SIC) approach prior to understanding the message in the downlink NOMA. This method aims to cancel out co-channel impedance from users on feebler channels [20].

Additionally, NOMA still needs to lower the energy required for the "amplify and forward" (AF) process to serve as a 5G basis. To remedy NOMA's inadequacies, a technology other than 5G is therefore required. The authors were motivated by this to create NOMA, which serves as the foundation for the 5G system that implements RIS-aided NOMA and realizes the ideas of the 6G system. [21], [22]. The feasible sum rate and outage performance for a downlink NOMA framework are covered in the reference [13]. The capacity to serve several customers at the same time, frequency, and code with varying degrees of power is the main benefit of NOMA over traditional OMA [23]-[25].

Many different channel types make up a Nakagami-\( m \) fading channel, with the Gaussian and Rayleigh fading channels serving as special cases. [26], [27]. Men, Ge, and Zhang [28] examined the performance of an AF relaying channels serving as special cases. [26], [27]. Gradshteyn and Ryzhik [31] were used to analyze the channel gains across Nakagami-\( m \) fading channels higher single rates than OMA [32], [33]. The majority of cooperative NOMA investigations up to this point have been conducted over Rayleigh fading channels in the p-CSI state. However, the channel estimate errors make them challenging to apply in practical wireless systems. Consequently, the authors proposed a study that is anticipated to contribute the following:

- The downlink system in NOMA supported by RIS can offer a reduced probability of blackout than conventional NOMA, according to the study's model scenario.
- Closed-form outage probability estimates for the RIS-assisted NOMA system are created. Since they are defined in terms of a wide range of various system parameters, it is possible to mathematically analyze how each system parameter affects the probability of an outage. For occurrence, the effect of the number of meta-surfaces in a RIS on the likelihood of an outage can be analyzed to improve the system's performance in actual operation. This study demonstrates that the number of meta-surfaces in RIS significantly impacts the system's outage probability.

II. MATERIALS AND METHOD

As seen in Fig. 1, we propose a two-user NOMA downlink based on RIS. A set of user groups is divided using orthogonal access. We suppose that each group contains representative users, such as a near-user (\( U_1 \)) and a far-user (\( U_2 \)), who are categorized according to their geographic location. The BS generates two beamforming vectors using the zero-force beamforming technique to serve two NOMA users. RIS-NOMA is helpful for developing various services since it can accommodate a wide range of Quality of Service (QoS) needs by grouping paired users. Once the user relies on the direct link connected to the BS, it becomes difficult.

Users \( U_1 \) and \( U_2 \) receive signals that are described by equations (1) and (2), respectively.

\[
y_{u_1} = |\sum_{n=1}^{N} h_n |g_{u_1}|^2| x_1 + \omega_{u_1} + n_{u_1} \tag{1}
\]

\[
y_{u_2} = |h_{d_2}| \sqrt{P_{x_2}} + \omega_{u_2} + n_{u_2} \tag{2}
\]

Where, \( \omega_{u_1} \) is interference term from \( U_1 \) with \( CN(0, \sigma^2) \) and \( \theta_{li} \) is the additional noise terms, which are interference signals from outside sources that can be considered AWGN noise when combined with \( CN(0, \Omega) \). The complex Gaussian channel vector terms for links RIS-U1 and BS-RIS, respectively, are denoted by the symbols \( g_{li} \) and \( h_i \). User \( U_2 \) and the BS follow Rayleigh fading, while \( h_{d2} \) is the corresponding fading channel between them. The matrix \( \theta_i (i = N) \) contains diagonal elements \( \exp(-j\theta_i) \) with \( \theta_i \) standing for the reflection phase shift.

First, by performing SIC, the SINR of \( U_1 \) to detect \( x_1 \) is given as mentioned in Eq. (3), where, \( \rho_1 = \frac{P_{u_1}}{\omega} \). Then, the SINR at \( U_1 \) to detect \( x_2 \) is given as Eq. (4). U2 receives direct signal \( x_2 \) from Base Station (BS). The SINR of \( U_2 \) to detect \( x_2 \) is given as Eq. (5).

\[
SINR(u_1, x_1, x_2) = \frac{\rho_1 |g_{u_1}|^2 a_{ps} \rho_2}{\sum_{n=1}^{N} |h_n| |g_{u_1}|^2 \rho_1 a_{ps} + \frac{n_{u_1}}{N} \left( d_{u_1} \right) \frac{n_{u_2}}{N} \left( d_{u_2} \right) \rho_1 + 1} \tag{3}
\]

\[
SINR(u_1, x_2) = \frac{\rho_1 |g_{u_1}|^2 a_{ps} \rho_2}{\sum_{n=1}^{N} |h_n| |g_{u_1}|^2 \rho_1 a_{ps} + \frac{n_{u_1}}{N} \left( d_{u_1} \right) \frac{n_{u_2}}{N} \left( d_{u_2} \right) \rho_1 + 1} \tag{4}
\]

\[
SINR(u_2, x_2) = \frac{|h_{d_2}|^2 a_{ps} \rho_2}{\rho_2 a_{ps} + \frac{n_{u_2}}{N} \left( d_{u_2} \right) \rho_2 + 1} \tag{5}
\]

As shown in Fig. 1, a link is defined as the BS transmitting a signal to U1 via RIS. Two channels are available on this link, one from BS to RIS and the other from RIS to U1. The \( U_1 \) is one user who receives a signal from BS via one of the
channels. The notation denotes the fading channel from RIS to U1|g_{11}, while the channel's fading coefficient of BS to RIS is represented by h_{11}. Additionally, other links involve the BS communicating with U2 via h_{22}.

By using RIS, physically, the fading channel h_{11} and g_{11} are related by h_{11} = \Phi g_{11} relation, where \Phi is denoted as phase shift. Using RIS, the fading channels h_{11} and g_{11} are physically connected by the h_{11} = \Phi g_{11} relation, where \Phi stands for phase shift. The authors assume that the link is composed of several channels. Special instances of the Nakagami-m distribution's broad spectrum include the Gaussian channel and the Rayleigh channel. Therefore, it is assumed that each channel has a Nakagami-m distribution.

The general form of PDF for Nakagami-m distribution [34] for one channel function is mentioned by Eq.(6).

\[
 f_{|\chi|^2}(x) = \frac{m_{\chi} \Gamma(m)}{\Gamma(m)} x^{m_{\chi}-1} e^{-\frac{m_{\chi} x}{1+\eta}}
\]

where, \chi, \eta and m are path-loss coefficients, relative channel estimation error of channel and fading parameter, respectively. Because \Gamma(m) = (m-1)! then Eq. (6) could be rewritten as Eq. (7)

\[
 f_{|\chi|^2}(x) = \frac{m_{\chi} \Gamma(m)}{\Gamma(m)} x^{m_{\chi}-1} e^{-\frac{m_{\chi} x}{1+\eta}}
\]

Cumulative Distribution Function (CDF) could be obtained by integrating the PDF above and expressed by Eq. (8)

\[
 F_{|\chi|^2}(x) = 1 - e^{-\frac{m_{\chi} x}{1+\eta}} \sum_{i=0}^{m_{\chi}-1} \frac{(m_{\chi} x)^i}{i!} (1+\eta)^{i}
\]

Based on Eq. (7) and Eq. (8), we create the formula for calculating the connection outage probability for each user. Initially, let’s assume that \zeta_1 and \zeta_2 stand in for the proper U_1 and U_2 target levels. The two SNR thresholds, \rho_{\theta 1} and \rho_{\theta 2}, can be expressed as Equations. (9) and (10), respectively.

\[
 \rho_{\theta 1} = \left(\frac{2\zeta_2-1}{\rho_s}\right)
\]

\[
 \rho_{\theta 2} = \left(\frac{2\zeta_1-1}{\rho_s}\right)
\]

Equations (3) and (4) can be used to obtain it as \tau_1 and \tau_2, the first and second comparison parameters against the SNR threshold, resulting in the equations:

(Proof: see Appendix A.)

\[
 \tau_1 = \frac{\rho_{\theta 1} h_{11}}{(a_1-a_1 \rho_{\theta 1}) \rho_s}
\]

\[
 \tau_2 = \frac{\rho_{\theta 1} h_{11}}{(a_2-a_2 \rho_{\theta 1}) \rho_s}
\]

To achieve closed-form outage performance, a parameter \lambda_\delta is defined as a gain in the channel coefficient of link v brought about by RIS implementation. If the RIS parts show random shifts, the definition \lambda_\delta = |h_{11} + \Phi g_{11}| is applicable. Furthermore, we determine \mathbb{E}[X_v^2] = \mathbb{E}[(X_v + \omega_{12}^2]^2] and variance \text{Var}(X_v) are calculated in appendix A. Eq. (13) is used to express \omega_v as the gamma distribution fading shape factor for the link v large-scale fading channel.

\[
 m_v = \frac{\mathbb{E}[X_v^2]}{\text{Var}(X_v)}
\]

where m_1 and m_2 are the gamma distribution shape factors for the large-scale fading channel on U_1 and U_2, respectively, and \nu is the link index in this instance \epsilon \{1,2\}.

The following might be written for PDF and CDF by integrating RIS in the NOMA network.

\[
 f_{|\chi|^2}(\tau) = \frac{m_{\chi} \Gamma(m_{\chi})}{\Gamma(m_{\chi})} e^{-\frac{m_{\chi} \tau}{1+\eta}}
\]

\[
 F_{|\chi|^2}(\tau) = 1 - e^{-\frac{m_{\chi} \tau}{1+\eta}} \sum_{i=0}^{m_{\chi}-1} \frac{(m_{\chi} \tau)^i}{i!} (1+\eta)^{i}
\]

The additional events of the outage in this study take place at U_i. When U_i correctly decodes both the signal x_2 and its own signal x_1, Eq. (16) could be used to express the outage probability of U_i.

\[
 P_{U_i} = 1 - \Pr(y(U_i,x_2) > \rho_{\theta 1} x_1, y(U_i,x_1) > \rho_{\theta 2} x_1)
\]

Eq. (16) could be written as Equations. (9) and (10), respectively. Because each channel suffers attenuation and noise, the link BS \rightarrow U_1 through RIS, which decodes itself signal x_1 could be determined as a parameter \lambda_1 is mentioned as Eq. (17).

\[
 \lambda_1 = \frac{\omega_{BS} d_{BS}^{\lambda_1} \rho_{\theta 1} x_1}{\eta_{BS} d_{BS}^{\lambda_1} \rho_{\theta 1} x_1} + 1
\]

where \omega_{BS} denotes the noise and attenuation brought on by each user’s use of RIS-aided equipment. By defining a_1 + a_2 = 1 where a_2 > a_1 then Eq. (16) could be expressed as Eq. (18).

\[
 P_{U_i} = 1 - e^{-\delta_1 \sum_{i=0}^{m_{\chi}-1} \frac{(\delta_1 \tau)^i}{i!}}
\]

where, m_i, \tau_1, \tau_2, \lambda_1 and \delta_1 are shown in Eq. (19), Eq. (11), Eq. (12), Eq. (17), and Eq. (20), respectively.

\[
 m_1 = \frac{(2\zeta_2+2\nu) \rho_{\theta 1} \rho_{\theta 2} \rho_{\theta 2}^{\delta_1} (d_{BS}^{\lambda_1} \rho_{\theta 1} x_1)}{(\nu^2+2\nu) \rho_{\theta 1} \rho_{\theta 2} \rho_{\theta 2}^{\delta_2} (d_{BS}^{\lambda_2} \rho_{\theta 1} x_1)}
\]

\[
 \delta_1 = \frac{m_1}{1-\eta_{BS} \rho_{\theta 1} x_1}
\]

The outage probability at U_i could then be expressed in Eq. (21).

\[
 P_{U_i} = 1 - e^{-\frac{m_1}{(1-\eta_{BS} \rho_{\theta 1} x_1)} \sum_{i=0}^{m_{\chi}-1} \frac{(m_1 \tau)^i}{i!}}
\]

(Proof: See in Appendix A.)

Similar reasoning might be used to determine the likelihood of an outage at U_2. According to the system model, the received signals which are received by U_2 consist of direct signals from BS and rely on a signal from U_1. The relying signal from U_1 is the link BS \rightarrow U_1 through RIS, which decodes signal x_2 for U_2 which has and could be mentioned as Eq. (16) above. We give the symbols dash-line the system model for this link, and there is no received signal by x_2 from RIS. Furthermore, the direct signal from BS to U_2 that has SNR as Eq. (5).

The first of two occurrences that make up the outage probability at U_2 is the inability of U_1 to decode the signal x_2. In addition, in the second instance, U_1 is unable to decode its own signal x_1 from BS, while U_1 can positively decode the signal x_2. In light of these occurrences, the following equation can be used to express the U_2 outage probability.
\[ P_{U_2} = 1 - e^{-\left(\delta_1 r_2 + \delta_3 s_2 \right) \sum_{j=0}^{m_2-1} \sum_{k=0}^{j} \left( \delta_1 r_2 \right) \left( \delta_3 s_2 \right) \left( j+k \right) } \]  
\[ m_2 = \frac{\beta_{SU_2}^2 + 2\beta_{SU_2}^2 + \beta_{RU_2}^2 + \beta_{RU_2}^2 + 2\beta_{RU_2}^2 \delta_3 + \beta_{RU_2}^2 \delta_3 + \beta_{SU_2}^2 \delta_3 + \beta_{SU_2}^2 \delta_3 + \beta_{SU_2}^2 \delta_3 + \beta_{SU_2}^2 \delta_3} {\beta_{SU_2}^2 + 2\beta_{SU_2}^2 + \beta_{SU_2}^2 + \beta_{SU_2}^2 + 2\beta_{SU_2}^2 \delta_3 + \beta_{SU_2}^2 \delta_3 + \beta_{RU_2}^2 \delta_3 + \beta_{RU_2}^2 \delta_3 + \beta_{SU_2}^2 \delta_3 + \beta_{SU_2}^2 \delta_3} \]  
where, \( W_2 = \eta_{SU_2} \left( d_{SU_2}^{-3} + \frac{\nu_{SR}}{N} \nu_{SR} \right) \cdot \frac{\eta_{RU_2}}{N} \cdot \frac{d_{RU_2}^{-3}}{N} \)

By inputting \( r_2 \) in Eq. (12), \( \lambda_1 \) in Eq. (17), \( m_2 \) in Eq. (23), \( r_2 \) in Eq. (24), \( \lambda_1 \) in Eq. (25), \( \delta_1 \) in Eq. (26), and \( m_2 \) = 2 then the outage probability in Eq. (22) could be solved.

\[ \tau_4 = \frac{\lambda_{PR}_{U_2}}{(a_2 - a_1 \nu_{PR}_{U_2})} \]  
\[ \lambda_2 = \eta_{SU_2} \left( d_{SU_2}^{-3} \right) + 1 \]  
Next, the \( \delta_3 \) is stated as Eq. (26).

\[ \delta_3 = \frac{m_2}{N} \left( \frac{d_{SR}^{-3}}{1 - \nu_{SR}} \right) + \frac{d_{RU_2}^{-3}}{1 - \nu_{RU_2}} \]  

\( \tau_4 \) is fourth-comparison-parameter against to SNR threshold. The link's gamma distribution at \( U_1 \) and \( U_2 \) has scale factors \( \delta_1 \) and \( \delta_2 \).

(Proof: see in Appendix-B).

III. RESULT AND DISCUSSION

In this part, the outage probability is simulated using mathematical derivations, and the simulation is validated using a Monte-Carlo simulation. Table 1 specifies the simulation parameters that will be used in the numerical simulation. Furthermore, the large-scale fading coefficient [dB] at near-user and far-user locations is modeled by \( \beta_k \).

\[ \beta_k = G_t + G_r + 10 \chi_k \ln_{10} \left( \frac{d_k}{1m} \right) - 30 + z_k \]  

where, \( k \in \{ SU_1, SR, RU_1, SU_2, SR, RU_2, U_1, U_2 \} \) and \( z_k \) is the shadow fading. Additionally, simulations are produced by changing settings in the MATLAB programming language. Monte Carlo simulations are used to validate the exact expressions of the outage probability. The previous investigations’ findings were numerically supported by mapping the sites into the Cartesian coordinate system.

**TABLE I**

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power allocation coefficient</td>
<td>( a_1 = 0.2 ), ( a_2 = 0.8 )</td>
</tr>
<tr>
<td>Path-loss exponent</td>
<td>( \chi_k = 2 )</td>
</tr>
<tr>
<td>Relative-channel estimation error</td>
<td>( \eta_k = 1 \times 10^{-4} \sim 9 \times 10^{-4} )</td>
</tr>
<tr>
<td>Distance between two nodes</td>
<td>( d_{SU_1} = 0.04(50m), d_{SR} = d_{SU_1} )</td>
</tr>
<tr>
<td></td>
<td>( d_{RU_1} = 0.02, d_{SU_2} = 0.04, d_{ru_2} = 0.04 )</td>
</tr>
<tr>
<td></td>
<td>( d_{U_1U_2} = 1 - d_{SU_1} )</td>
</tr>
<tr>
<td>Transmit SNR</td>
<td>( \rho_s = 0 \sim 60dB )</td>
</tr>
<tr>
<td>The antenna gains at transmitter and receiver</td>
<td>( G_t = 3.2d_B, G_r = 1.3dB )</td>
</tr>
<tr>
<td>Target Rate</td>
<td>( \zeta_1 = 3.6 \text{ BPCU}, \zeta_2 = 1 \text{ BPCU} )</td>
</tr>
</tbody>
</table>

For user \( U_1 \), Fig. 2 displays the simulation and analytical findings of the outage performance against SNR while using the RIS on the NOMA network for various RIS element counts. The outage performance versus SNR by NOMA Network is also compared. The graph clearly shows that as the number of RIS pieces rises, the probability of an outage falls. The NOMA network's RIS implementation performs better during outages after the installation of RIS components. Additionally, it performs better than the NOMA outage performance of the network system. This demonstrates that the suggested RIS-NOMA system performs effectively, as predicted by our study.

![Fig. 2 The Outage probability for U1](image)

![Fig. 3 The Outage probability for U2](image)

![Fig. 4 Comparison of the Outage probability between U1 and U2](image)
Furthermore, it is evident from the aforementioned two figures that $U_1$’s outage probability outperforms that of $U_2$. Fig. 4 compares the outage probability versus SNR for various numbers of RIS elements for near-user $U_1$ and far-user $U_2$ based on simulation and analytical results. In this comparison, it is clear that the near-user $U_1$ performs significantly better than the far-user $U_2$ as the number of RIS elements grows. It proves that even without amplification and forward (AF) mode, the RIS implementation on the NOMA network can outperform the NOMA network in terms of outage performance. The only mode utilized is decoded and forward (DF).

IV. CONCLUSION

In this study, we deduced how well a RIS-NOMA system performed during an outage. The closed-form equation was discovered for the probability of an outage for users $U_1$ without a direct link and $U_2$ with a direct link. We simplified the system performance study on the gap between two users by assuming Nakagami - m with p-CSI, and this paper only focuses on the major performance parameter, outage probability. To confirm the correctness of our formulas, Monte Carlo simulations are performed. Future development will take multiple users at the RIS-NOMA system into consideration.

REFERENCES


It is described, \( X_{v} = \sum_{k=1}^{N} h_{k}^{v} g_{k}^{v} \). In the event that it is thought that the signal reflected by the RIS components contains random phase fluctuations. It is acceptable to suppose that \( \hat{h}_{n} \sim \mathcal{CN}(0, \beta_{SR} I_{N}), \) \( g_{k} \sim \mathcal{CN}(0, \beta_{R_{V},1} I_{N}) \) and \( \phi = \text{diag}(\theta_{B_{1}}, \ldots, \theta_{B_{N}}) \), where \( I_{N} \) and \( \theta_{B_{n}} \in [-\pi, \pi] \) are character matrices of order \( N \) with a range for \( \theta_{B_{n}} \) respectively, due to the identical independent distribution of each channel (i.i.d). In addition, we define the anticipated value. The independence of the channels that make up \( \mathbb{E}[X_{v}] \) are averaged. Eq could be used to express it (A.1).

\[
\mathbb{E}[X_{v}] = \mathbb{E}\left[h_{k}^{v} g_{k}^{v} \right] \tag{A.1}
\]

\( X_{v} \) can be written as an equation and represents the average link \( \hat{X}_{v} \) and fade due to path-loss distance-induced attenuation \( \sigma_{v}^{2} \) in the propagation location (A.2). Path-loss distance-induced attenuation \( \sigma_{v}^{2} \) could be defined as \( \frac{\mathbb{E}[X_{v}]}{N} = \left( \frac{d_{SR}^{\gamma}}{\mathbb{E}[g_{k}^{v}]} \right) \frac{\mathbb{E}[X_{v}]}{N} \left( \frac{d_{R_{V},1}^{\gamma}}{\mathbb{E}[g_{k}^{v}]} \right) \), then Eq. (A.2) might be revised as Eq. (A.3).

\[
\mathbb{E}[X_{v}] = \mathbb{E}\left[h_{k}^{v} g_{k}^{v} \right] + \mathbb{E}[X_{v}] = \mathbb{E}[X_{v}] + \mathbb{E}[X_{v}] \tag{A.2}
\]

\[
\mathbb{E}[X_{v}] = \mathbb{E}\left[h_{k}^{v} g_{k}^{v} \right] + \mathbb{E}[X_{v}] = \mathbb{E}[X_{v}] + \mathbb{E}[X_{v}] \tag{A.3}
\]

We define that \( \mathbb{E}\left[h_{k}^{v} g_{k}^{v} \right] = N_{SR} \beta_{R_{V},1} \), so that Eq. (A.3) could be written as Eq. (A.4).

\[
\mathbb{E}[X_{v}] = N_{SR} \beta_{R_{V},1} + \mathbb{E}[X_{v}] = \mathbb{E}[X_{v}] + \mathbb{E}[X_{v}] \tag{A.4}
\]

Furthermore, it is determined \( \mathbb{E}\left[h_{k}^{v} g_{k}^{v} \right] \) as Eq. (A.5) below.

\[
\mathbb{E}[X_{v}] = \mathbb{E}\left[h_{k}^{v} g_{k}^{v} \right] \tag{A.5}
\]

Using the circular symmetric properties, Eq. (A.5) could be written as Eq. (A.6).

\[
\mathbb{E}[X_{v}] = \mathbb{E}\left[h_{k}^{v} g_{k}^{v} \right] \tag{A.6}
\]

If it is defined \( z = \begin{bmatrix} j_{0}^{v} g_{0}^{v} \\ \vdots \\ j_{N}^{v} g_{N}^{v} \end{bmatrix} \sim \mathcal{CN}(0, \beta_{SR}) \), then

\[
\mathbb{E}[X_{v}] = \mathbb{E}\left[h_{k}^{v} g_{k}^{v} \right] \tag{A.7}
\]

We could get Eq. (A.24) by entering the values \( \mathbb{E}\left[h_{k}^{v} g_{k}^{v} \right] \) into Eq. (A.6). It can also be described as \( X_{v} = \hat{X}_{v} + e_{v} = \hat{X}_{v} + \sigma_{v}^{2} \), allowing the expected-value of \( X_{v} \) to express in the form of Eq. (A.8).

\[
X_{v} = N(\hat{X}_{v} + e_{v}) \tag{A.8}
\]

Based on (A.8), we specify \( \mathbb{E}[X_{v}] = \mathbb{E}\left[X_{v} \right] + \sigma_{v}^{2} \) and could be rewritten as Eq. (A.9) ad Eq. (A.10).

\[
\mathbb{E}[X_{v}] = \mathbb{E}\left[h_{k}^{v} g_{k}^{v} \right] + \frac{\mathbb{E}[X_{v}]}{N} \left( \frac{d_{SR}^{\gamma}}{\mathbb{E}[g_{k}^{v}]} \right) \frac{\mathbb{E}[X_{v}]}{N} \left( \frac{d_{R_{V},1}^{\gamma}}{\mathbb{E}[g_{k}^{v}]} \right) \tag{A.9}
\]
According to the equation of $\gamma_{U_2,BS'}$ in the Eq. (B.1).

$$\gamma_{U_2,BS} = \frac{|h_{U_2}|^2a_2\rho_x}{a_1\rho_x\eta_{SU_2}(d_{SU_2}^X\rho_x + 1)}$$  \hspace{1cm} (B.1)

We derive $\tau_4$ base on the system model, which could be conducted as follows.

If it is defined $B = |h_{D_2}|^2$ and $\lambda_3 = \eta_{SU_2}(d_{SU_2}^X\rho_x + 1)$, then

$$\Rightarrow \frac{|h_{D_2}|^2a_2\rho_x}{a_1\rho_x\eta_{SU_2}(d_{SU_2}^X\rho_x + 1)} \geq \rho_{TH2} \hspace{1cm} (B.2)$$

$$\Rightarrow \frac{B\rho_x}{a_1\rho_x + \lambda_3} \geq \rho_{TH2} \hspace{1cm} (B.3)$$

$$\Rightarrow B\rho_x \geq (B a_1\rho_x + \lambda_3)\rho_{TH2} \hspace{1cm} (B.4)$$

$$\Rightarrow B\rho_x - B a_1\rho_x \rho_{TH2} \geq \lambda_3\rho_{TH2} \hspace{1cm} (B.5)$$

$$\Rightarrow B \geq \frac{\lambda_3\rho_{TH2}}{(a_2-a_1\rho_x)\rho_x} \hspace{1cm} (B.6)$$

$$\Rightarrow \tau_4 = \frac{\lambda_3\rho_{TH2}}{(a_2-a_1\rho_x)\rho_x} \hspace{1cm} (B.8)$$

The outage probability at user $U_2$ could be shown as follows.

$$P_{U_2} = Pr(\gamma_{U_2,BS} < \gamma_{TH2}) + Pr(\gamma_{U_2,BS} > \gamma_{TH2}) \hspace{1cm} (B.9)$$

$$P_{U_2} = Pr(\gamma_{U_2,BS} > \gamma_{TH2}) Pr(\gamma_{U_2,BS} > \gamma_{TH2}) \hspace{1cm} (B.10)$$

$$P_{U_2} = 1 - Pr(\gamma_{U_2,BS} > \gamma_{TH2}) Pr(\gamma_{U_2,BS} > \gamma_{TH2}) \hspace{1cm} (B.11)$$

$$P_{U_2} = 1 - e^{-\delta_1\tau_2} \sum_{j=0}^{m_1-1} (\delta_1\tau_2)^j/j! e^{-\delta_3\tau_4} \sum_{k=0}^{m_2-1} (\delta_3\tau_4)^k/k! \hspace{1cm} (B.12)$$

**APPENDIX B**

**PROOF OF PROPOSITION**

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_{U_2,BS}$</td>
<td>The RIS Reflected Signal's Amplitude</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Coefficient with $\alpha \in [0,1]$</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>Phase modification is possible with the $i$-th reflected element of RIS.</td>
</tr>
<tr>
<td>$\phi$</td>
<td>The phase-shift matrix, $\text{diag}(\exp(j\theta_1), \exp(j\theta_2), \ldots, \exp(j\theta_N))$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Hermitean transpose</td>
</tr>
<tr>
<td>$\Omega_k$</td>
<td>The large-scale fading coefficients of the channel $k$</td>
</tr>
<tr>
<td>$\Delta_k$</td>
<td>The link power of channel $k$</td>
</tr>
<tr>
<td>$\hat{h}_k$</td>
<td>Average-connection power of channel $k$</td>
</tr>
<tr>
<td>$h_k$</td>
<td>Average-fading coefficient of channel $k$</td>
</tr>
<tr>
<td>$\beta_k$</td>
<td>Fading-coefficient of channel $k$</td>
</tr>
<tr>
<td>$\hat{X}_0$</td>
<td>Average fading-coefficient gain by RIS of $v$ user</td>
</tr>
<tr>
<td>$X_v$</td>
<td>Gain in fading coefficient by $v$ user's RIS</td>
</tr>
<tr>
<td>$\sigma_v^2$</td>
<td>Channel estimation error</td>
</tr>
<tr>
<td>$\eta_k$</td>
<td>Channel $k$ with relative channel estimation error</td>
</tr>
<tr>
<td>$m_v$</td>
<td>The gamma distribution channel's shape factor</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Path-loss exponent</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>Distance of two-points crossed by channel $k$</td>
</tr>
<tr>
<td>$P_{\text{c}}$</td>
<td>Outage probability at $(\cdot)$</td>
</tr>
<tr>
<td>$n_u$ and $h_u$</td>
<td>AWGN at $U_1$ and $U_2$</td>
</tr>
<tr>
<td>$d_{SU_1},d_{SU_2},d_{AU_1},d_{AU_2}$</td>
<td>Distance of $BS - U_1$, $BS - RIS$, and $RIS - U_1$, respectively</td>
</tr>
<tr>
<td>$\theta_{BS_1},\theta_{BS_2},\theta_{RIS}$</td>
<td>Distance of $BS - U_2$, $BS - RIS$, and $RIS - U_2$, respectively</td>
</tr>
<tr>
<td>$\psi_0$</td>
<td>Coefficients of fading channel</td>
</tr>
<tr>
<td>$\rho_{\text{SNR}}$</td>
<td>Transmit signal-to-noise ratio (SNR)</td>
</tr>
<tr>
<td>$\rho_{U_1-BS}$</td>
<td>At $U_1$, examine the incoming signal for interference and noise ratio in order to decode it (SNR).</td>
</tr>
<tr>
<td>$\rho_{U_2}$</td>
<td>The received SINR of $U_1$ and $U_2$ to decode the signal itself.</td>
</tr>
<tr>
<td>$\rho_{2,1}$</td>
<td>The received SINR of $U_2$ to decode the signal $s_t$ to relay link $\rho_{0,1}$</td>
</tr>
<tr>
<td>$\rho_{SC}$</td>
<td>The received SINR after selection combining (SC) at $U_2$</td>
</tr>
<tr>
<td>$\rho_{TH2}$</td>
<td>SINR target of $U_1$ and $U_2$</td>
</tr>
<tr>
<td>$R_1$ and $R_2$</td>
<td>Target rate of user $U_1$ and $U_2$</td>
</tr>
<tr>
<td>$P_{U_1}$ and $P_{U_2}$</td>
<td>Outage probability of $U_1$ and $U_2$</td>
</tr>
<tr>
<td>$P_{\text{c}}$</td>
<td>The first-comparison parameter, the second-comparison parameter, and the third-comparison parameter are all variables.</td>
</tr>
<tr>
<td>$\lambda_1$, $\lambda_2$</td>
<td>Interference and noise due to the using of RIS-aided of $U_1$ and $U_2$</td>
</tr>
<tr>
<td>$\delta_1$, $\delta_2$</td>
<td>Scale factor of the gamma-distribution of the channel of $U_1$ and $U_2$</td>
</tr>
</tbody>
</table>

**NOTATION**

**TABLE OF NOTATION**

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>$s(t)$</td>
<td>A superimposed signal is sent to both the near $(U_i)$ and far $(U_j)$ users.</td>
</tr>
<tr>
<td>$P_s$</td>
<td>Transmitted signal power</td>
</tr>
<tr>
<td>$P_r$</td>
<td>Transmitted signal power by $U_j$</td>
</tr>
<tr>
<td>$a_2$ and $a_2$</td>
<td>Power level $s_t$ and $s_2$</td>
</tr>
<tr>
<td>$N$</td>
<td>The quantity of RIS components</td>
</tr>
</tbody>
</table>