APPENDIX A
PROOF OF PROPOSITION

It is described, $X_v = |\sum_{l=1}^{N} h_{vl}^H \Phi g_{vl}|^2$. In the event that it is thought that the signal reflected by the RIS components contains random phase fluctuations. It is acceptable to suppose that $\hat{h}_l \sim \mathbb{CN}(0, \beta_{SR} I_N)$, $\hat{g}_{vl} \sim \mathbb{CN}(0, \beta_{RU_v} I_N)$and $\phi = diag(e^{j\theta_1}, e^{j\theta_2}, \ldots, e^{j\theta_N})$, where I_N and $\theta_n \in [-\pi, \pi]$are character matrices of order N with a range for θ_n, respectively, due to the identical independent distribution of each channel (i.i.d). In addition, we define the anticipated value. The independence of the channels that make up $\mathbb{E}\{|X_v|\}$ are averaged. Eq could be used to express it (A.1).

$$\mathbb{E}\{|X_v|\} = \mathbb{E}\left\{\left|h_l^H \Phi g_{vl}\right|^2\right\} \quad (A.1)$$

$X_v$can be written as an equation and represents the average link $\hat{X}_v$ and fade due to path-loss distance-induced attenuation $\sigma_{e_v}^2$ in the propagation location (A.2).

$$\mathbb{E}\{|X_v|\} = \mathbb{E}\{\hat{X}_v + \sigma_{e_v}^2\} = \mathbb{E}\{\hat{X}_v\} + \sigma_{e_v}^2 \quad (A.2)$$

Path-loss distance-induced attenuation $\sigma_{e_v}^2$ could be defined as $\frac{\eta_{SR}}{N}(d_{SR}^{-\chi}).\frac{\eta_{RU_v}}{N}(d_{RU_v}^{-\chi})$, then Eq. (A.2) might be revised as Eq. (A.3).

$$\mathbb{E}\{|X_v|\} = \mathbb{E}\left\{\left|\hat{h}_l^H \Phi \hat{g}_{vl}\right|^2\right\} + \frac{\eta_{SR}}{N}(d_{SR}^{-\chi}).\frac{\eta_{RU_v}}{N}(d_{RU_v}^{-\chi}) \quad (A.3)$$

We define that $\mathbb{E}\left\{\left|\hat{h}_l^H \Phi \hat{g}_{vl}\right|^2\right\} = N\beta_{SR}\beta_{RU_v}$, so that Eq. (A.3) could be written as Eq. (A.4)

$$\mathbb{E}\{|X_v|\} = N\beta_{SR}\beta_{RU_v} + \frac{\eta_{SR}}{N}(d_{SR}^{-\chi}).\frac{\eta_{RU_v}}{N}(d_{RU_v}^{-\chi}) \quad (A.4)$$

Furthermore, it is determined $\mathbb{E}\left\{\left|\hat{X}_v\right|^2\right\}$as Eq. (A.5) below.

$$\mathbb{E}\left\{\left|\hat{X}_v\right|^2\right\} = \mathbb{E}\left\{\left|\hat{h}_l^H \Phi \hat{g}_{vl}\right|^2\right\} \quad (A.5)$$

Using the circular symmetric properties, Eq. (A.5) could be written as Eq. (A.6).

$$\mathbb{E}\left\{\left|\hat{X}_v\right|^2\right\} = \mathbb{E}\left\{||\Phi \hat{g}_{vl}||^4 \left|\frac{\hat{h}_l^H \Phi \hat{g}_{vl}}{||\Phi \hat{g}_{vl}||}\frac{\hat{g}_{vl}^H \Phi^H \hat{h}_l}{||\Phi \hat{g}_{vl}||}\right|^2\right\} \quad (A.6)$$

If it is defined $z = \frac{\hat{h}_l^H \Phi \hat{g}_{vl}}{||\Phi \hat{g}_{vl}||}; z \sim CN(0, \beta_{SR})$, then

$$\mathbb{E}\left\{\left|\hat{X}_v\right|^2\right\} = \mathbb{E}\left\{||\Phi \hat{g}_{vl}||^4 |z|^4\right\} = \mathbb{E}\left\{||\Phi \hat{g}_{vl}||^4\right\}\mathbb{E}\{|z|^4\} \quad (A.7)$$

We could get Eq. (A.24) by entering the values $\mathbb{E}\left\{||\Phi \hat{g}_{vl}||^4\right\}\mathbb{E}\{|z|^4\}$ into Eq (A.6).

It can also be described as $X_v = \hat{X}_v + e_v = \hat{X}_v + \sigma_{e_v}^2$, allowing the expected-value of ${X_v}^2$ to express in the form of Eq. (A.8).

$$\mathbb{E}\left\{\left|\hat{X}_v\right|^2\right\} = N(N+1)\beta_{RU_v}^2 2\beta_{SR}^2 = (2N^2 + 2N)\beta_{RU_v)}^2\beta_{SR}^2 \quad (A.8)$$

Based on Eq. (A.8), we specify $\mathbb{E}\{|X_v|^2\} = \mathbb{E}\left\{\left|\hat{X}_v\right|^2\right\} + \sigma_{e_v}^2$and could be rewritten as Eq. (A.9) ad Eq. (A.10).

$$\mathbb{E}\{|X_v|^2\} = \mathbb{E}\left\{\left|\left|\hat{h}_l^H \Phi \hat{g}_{vl}\right|^2 + \left|\frac{\eta_{SR}}{N}(d_{SR}^{-\chi}).\frac{\eta_{RU_v}}{N}(d_{RU_v}^{-\chi})\right|\right|^2\right\} \quad (A.9)$$

$$\mathbb{E}\{|X_v|^2\} = (2N^2 + 2N)\beta_{RU_v}^2\beta_{SR}^2 + \left|\frac{\eta_{SR}}{N}(d_{SR}^{-\chi}).\frac{\eta_{RU_v}}{N}(d_{RU_v}^{-\chi})\right|^2 \quad (A.10)$$

By defining, $Var\{|X_v|\} = E\{|X_v|^2\} - |E\{|X_v|\}|^2$, then it may be said to be as Eq.(A.11).

$$Var\{|X_v|\} = (N^2 + 2N)\beta_{RU_v}^2\beta_{SR}^2 - 2N\beta_{SR}\beta_{RU_v}\frac{\eta_{SR}}{N}(d_{SR}^{-\chi}).\frac{\eta_{RU_v}}{N}(d_{RU_v}^{-\chi}) \quad (A.11)$$

Next,$\tau_1$in Eq. (11) could be derived by defining ρ_{Th_1}as Eq. (A.12).

$$\rho_{Th_1} = \frac{(2^{2\zeta_1} - 1)}{\rho_s} \quad (A.12)$$

From Eq. (7), we define the notation $A_1 = \left|\sum_{l=1}^{L}|\hat{h}_l||\hat{g}_{1l}|\right|^2and\lambda_1 = \frac{\eta_{SR}}{N}(d_{SR}^{-\chi}).\frac{\eta_{RU_1}}{N}(d_{RU_1}^{-\chi})\rho_s + 1$ so that it can be rewritten Eq. (7) as Eq. (A.18).

$$\frac{A_1 a_1 \rho_s}{A_1 a_1 \rho_s + \lambda_1} \geq \rho_{Th_1} \quad (A.13)$$

$$\Rightarrow A_1 a_1 \rho_s \geq (A_1 a_1 \rho_s + \lambda_1)\rho_{Th_1} \quad (A.14)$$

$$\Rightarrow A_1 a_1 \rho_s - A_1 a_1 \rho_s \rho_{Th_1} \geq \lambda_1 \rho_{Th_1} \quad (A.15)$$

$$\Rightarrow A_1 \rho_s (a_1 - a_1 \rho_{Th_1}) \geq \lambda_1 \rho_{Th_1} \quad (A.16)$$

$$\Rightarrow A_1 \geq \frac{\lambda_1 \rho_{Th_1}}{(a_1 - a_1 \rho_{Th_1})\rho_s} \quad (A.17)$$

$$\Rightarrow \tau_1 = \frac{\lambda_1 \rho_{Th_1}}{(a_1 - a_1 \rho_{Th_1})\rho_s} \quad (A.18)$$

Similarly, it is defined ρ_{Th_2}in Eq. (A.19).

$$\rho_{Th_2} = \frac{(2^{2\zeta_2} - 1)}{\rho_s} \quad (A.19)$$

then, $\tau_2$could be derived from Eq. (A.19) as follows.

$$\frac{A_2 a_2 \rho_s}{A_2 a_1 \rho_s + \lambda_1} \geq \rho_{Th_2} \quad (A.20)$$

$$\Rightarrow A_2 a_2 \rho_s \geq (A_2 a_1 \rho_s + \lambda_1)\rho_{Th_2} \quad (A.21)$$

$$\Rightarrow A_2 a_2 \rho_s - A_2 a_1 \rho_s \rho_{Th_2} \geq \lambda_1 \rho_{Th_2} \quad (A.22)$$

$$\Rightarrow A_2 \rho_s (a_2 - a_1 \rho_{Th_2}) \geq \lambda_1 \rho_{Th_2} \quad (A.23)$$

$$\Rightarrow A_2 \geq \frac{\lambda_1 \rho_{Th_2}}{(a_2 - a_1 \rho_{Th_2})\rho_s} \quad (A.24)$$

$$\Rightarrow \tau_2 = \frac{\lambda_1 \rho_{Th_2}}{(a_2 - a_1 \rho_{Th_2})\rho_s} \quad (A.25)$$

The outage probability at user U_1 could be mentioned in Eq. (A.26), which is determined by making changes to the equations of PDF and CDF given above as a result of the deployment of RIS in NOMA networks.

$$P_{U_1} = 1 - Pr\left(\left|\hat{X}_1\right|^2 > \tau_2, \left|\hat{X}_1\right|^2 > \tau_1\right) \quad (A.26)$$

so that, by rearranging Eq. (A.26) obtained as Eq. (A.27).

$$P_{U_1} = 1 - Pr\left(\left|\hat{X}_1\right|^2 > max(\tau_2, \tau_1)\right) \quad (A.27)$$

$$P_{U_1} = 1 - \left(\left|\hat{X}_1\right|^2 > \tau\right) \quad (A.28)$$

where, according to which value is the highest, τ might be as high as $\tau_2$orτ_1. The reason for this is that there is a higher probability that U_1 will not successfully decode the signal x_2 from another user, U_2, or its signal, x_2, than that any of those two events will occur. Consequently, Eq. (A.27) is written as Eq. (A.28).

APPENDIX B
PROOF OF PROPOSITION

According to the equation of $\gamma_{U_{2,BS}}$, in the Eq. (B.1).

$$\gamma_{U_{2,BS}} = \frac{|h_{D_2}|^2 a_2\rho_s}{a_1\rho_s+\eta_{SU_2}\, d_{SU_2}^{-\chi}\rho_s+1} \quad \text{(B.1)}$$

We derive τ_4 base on the system model, which could be conducted as follows.

If it is defined $B = |\hat{h}_{D_2}|^2$ and $\lambda_3 = \eta_{SU_2}\left(d_{SU_2}^{-\chi}\right)\rho_s + 1$, then

$$\Rightarrow \frac{|\hat{h}_{D_2}|^2 a_2\rho_s}{|\hat{h}_{D_2}|^2 a_1\rho_s+\eta_{SU_2}\left(d_{SU_2}^{-\chi}\right)\rho_s+1} \geq \rho_{Th_2} \quad \text{(B.2)}$$

$$\Rightarrow \frac{Ba_2\rho_s}{Ba_1\rho_s+\lambda_3} \geq \rho_{Th_2} \quad \text{(B.3)}$$

$$\Rightarrow Ba_2\rho_s \geq (Ba_1\rho_s + \lambda_3)\rho_{Th_2} \quad \text{(B.4)}$$

$$\Rightarrow Ba_2\rho_s - Ba_1\rho_s\rho_{Th_2} \geq \lambda_3\rho_{Th_2} \quad \text{(B.5)}$$

$$\Rightarrow B\rho_s\left(a_2 - a_1\rho_{Th_2}\right) \geq \lambda_3\rho_{Th_2} \quad \text{(B.6)}$$

$$\Rightarrow B \geq \frac{\lambda_3\rho_{Th_2}}{(a_2-a_1\rho_{Th_2})\rho_s} \quad \text{(B.7)}$$

$$\Rightarrow \tau_4 = \frac{\lambda_3\rho_{Th_2}}{(a_2-a_1\rho_{Th_2})\rho_s} \quad \text{(B.8)}$$

The outage probability at user U_2 could be shown as follows.

$$P_{U_2} = Pr\left(\gamma_{U_{2,BS}} < \gamma_{Th_2}\right) + Pr\left(\gamma_{U_{2,BS}} > \gamma_{Th_2}, \gamma_{U_{2,U_1}} < \gamma_{Th_2}\right) \quad \text{(B.9)}$$

$$P_{U_2} = Pr\left(\gamma_{U_{2,BS}} < \gamma_{Th_2}\right) + Pr\left(\gamma_{U_{2,BS}} > \gamma_{Th_2}\right) Pr\left(\gamma_{U_{2,U_1}} < \gamma_{Th_2}\right) \quad \text{(B.10)}$$

$$P_{U_2} = 1 - Pr\left(\gamma_{U_{2,BS}} > \gamma_{Th_2}\right) Pr\left(\gamma_{U_{2,U_1}} > \gamma_{Th_2}\right) \quad \text{(B.11)}$$

$$P_{U_2} = 1 - e^{-\delta_1\tau_2}\sum_{j-0}^{m_1-1}\frac{(\delta_1\tau_2)^j}{j!} e^{-\delta_3\tau_4}\sum_{k=0}^{m_3-1}\frac{(\delta_3\tau_4)^k}{k!} \quad \text{(B.12)}$$

NOTATION

TABLE OF NOTATION [35].

Notation	Definition
$s(t)$	A superimposed signal is sent to both the near (U_1) and far (U_2) users.
P_s	Transmitted signal power
P_1	Transmitted signal power by U_1
a_1 and a_2	Power level x_1 and x_2
N	The quantity of RIS components
α	The RIS Reflected Signal's Amplitude Coefficient with $\alpha \in (0,1]$
θ_l	Phase modification is possible with the *l-th* reflected element of RIS.
Φ	The *phase-shift matrix*, diag(exp(jθ_1), exp(jθ_2), …, exp(jθ_L))
$(.)^H$	Hermitian transpose
β_k	The large-scale fading coefficients of the channel *k*
Ω_k	The link power of channel *k*
$\hat{\Omega}_k$	Average-connection power of channel *k*
$\hat{h}_k$	Average-fading coefficient of channel *k*
h_k	Fading-coefficient of channel *k*
$\hat{X}_v$	Average fading-coefficient gain by RIS of *v* user
X_v	Gain in fading coefficient by *v* user's RIS
e_k	Channel estimation error
$\sigma_{e_k}^2$	Channel estimation error variant
η_k	Channel k with relative channel estimation error
m_v	The gamma distribution channel's shape factor
χ	Path-loss exponent
d_k	Distance of two-points crossed by channel *k*
$P_{(..)}$	Outage probability at (..)
n_{U_1} and n_{U_2}	AWGN at U_1 and U_2.
$d_{SU_1}, d_{SR_1}, d_{RU_1}$	Distance of BS − U_1, BS − RIS, and RIS − U_1
$d_{SU_2}, d_{SR_2}, d_{RU_2}$	Distance of BS − U_2, BS − RIS, and RIS − U_2
$h_{D_1}, h_{D_2}, h_l, g_{1l}, g_{2l}$	Coefficients of fading channel
ρ_s	Transmit signal-to-noise ratio (SNR)
$\rho_{U_2\to U_1}$	At U_1, examine the incoming signal for interference and noise ratio in order to decode it (SINR).
ρ_{U_1}, ρ_{U_2}	The received SINR of U_1 and U_2 to decode the signal itself.
$\rho_{2,U2}$	The received SINR of U_2 to decode the signal x_2 to relay link
$\rho_{U_2}^{SC}$	The received SINR after selection combining (SC) at U_2
ρ_{Th_1} and ρ_{Th_2}	SINR target of user U_1 and U_2
R_1 and R_2	Target rate of user U_1 and U_2
P_{U_1} and P_{U_2}	Outage probability of U_1 and U_2
τ_1, τ_2 and τ_3	The first-comparison parameter, the second-comparison parameter, and the third-comparison parameter are all variables.
λ_1, λ_2	Interference and noise due to the using of RIS-aided of U_1 and U_2
δ_1, δ_2	Scale factor of the gamma-distribution of the channel of U_1 and U_2