JOIV : Int. J. Inform. Visualization, 7(1) - March 2023 168-177

INTERNATIONAL JOURNAL
ON INFORMATICS VISUALIZATION

journal homepage : www.joiv.org/index.php/joiv

The Joint Decision-Making Support through Piecewise Objective Optimization Model for Integrated Supplier Selection, Inventory Management, and Production Planning Involving Discounts

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***Abstract*—The decision-makers in manufacturing industries continuously optimize every supply-chain part to achieve optimal profit. In this paper, three crucial activities in the supply chain are observed as profit contributors: supplier selection, inventory management, and production planning. Decision-making support is needed to optimize those activities, especially when prices/costs involve discounts. Therefore, this study aims to develop integrated decision-making support for supplier selection, inventory management, and production planning involving discounted prices. The problem was considered with multi-supplier, multi-raw material, multi-product, and multi-observation time instant. The objective was based on maximizing the profit for the entire activity, i.e., from the raw material procurement and storage to the production. This supply chain was modeled as mixed-integer linear programming with a piecewise objective function representing the profit, which was maximized. It was also modeled with a bunch of constraint functions, including product demand satisfaction. The proposed model was tested with computational simulations using randomly generated supply chain data. The primal simplex algorithm was also employed to calculate the real value of the optimal decision, which was combined with the Branch-and-Bound approach to calculate the appropriate integer solution. The results showed that the optimal decision was achieved, namely (1) The optimal quantity of raw materials ordered to each supplier, (2) The optimal production quantity, and (3) The optimal inventory level, which provided the maximal profit for the whole optimization time horizon. This indicated that the proposed decision-making support model is implementable for industrial decision-makers.**

***Keywords*—Joint decision-making support; primal simplex; linear programming; supplier selection; integrated supply chain.**

Manuscript received 24 Jan. 2022; revised 19 Apr. 2022; accepted 24 Oct. 2022. Date of publication 31 Mar. 2023.

I. Introduction

Decision-makers in manufacturing industries often continuously optimize their operational activities to achieve maximal profit. This shows that a manufacturer plays a role in at least three parts of the supply chain: raw material procurement, production, and inventory. Raw material procurement deals with allocating goods to the suppliers, with the production segment determining the optimal quantity of the products. Meanwhile, the inventory phase determines the optimal quantity of the raw materials and products to be stored. The flow of these materials and products is also connected, indicating that the integrated optimization of the supply chain parts is very beneficial. The problem emphasizes discounted prices, where there are discounts for raw materials, transportation, inventory, and product costs. In a supply chain, a specific segment is often independently optimized, as several models have reportedly been proposed with different environments for the supplier selection problem. For example, two studies by Ware et al. [1], [2], for solving basic supplier selection problems with different cost functions, proposed simple linear and nonlinear programming models. With a more complex situation, such as facility disruption, a highly complicated model was needed, as proposed by Rafiei et al. [3]. Other special conditions also emphasized the problems with deteriorating products [4], [5], and fast service requirements [6]. In addition, the case studies regarding supplier selection problems were observed in many sectors, including automotive [7], banking [8], power plant management [9]–[11], logistics [12], [13], steel industries [14], etc.

The production planning problem has subsequently been solved using many approaches, for instance, linear programming [15] and nonlinear programming [16], based on the considered conditions, such as random parameters. According to studies by Yazdani et al. [17]–[20, sustainability problems could also be solved by implementing multi-objective optimization models. Moreover, many previous case studies emphasized the applicability of optimization models as the decision-making support for production planning in many manufacturing industries. These included chemical, gas, food production, sawmills, copper, textile, and dairy production company [21], [22], [23], [24], [25], [26], [27], [28].

TABLE I
RELATED WORKS; SS: SUPPLIER SELECTION, IM: INVENTORY MANAGEMENT, PP: PRODUCTION PLANNING

Source	SS	IM	PP	Integrated	Parameters			Multi Supp.	Multi Goods	Multi-period
					Cert.	Unc.	Disc.			
[29]	✓	×	×	×	×	✓	×	✓	×	✓
[30]	✓	×	×	×	×	✓	×	✓	×	×
[31]	✓	×	×	×	✓	×	×	✓	✓	×
[32]	✓	×	×	×		✓	×	✓	✓	✓
[33]	×	✓		×	✓	×	×	×	✓	✓
[34]	✓	✓	✓	SS-IM-PP	✓	×	×	✓	✓	✓
[35]	×	×	✓	×	✓	×	×	×	✓	✓
[36]	×	×	✓	×	✓		✓	×		✓
[37]	✓	✓	×	SS-IM	✓			✓	✓	✓
[38]	✓	✓	×	SS-IM	✓		✓	✓	✓	✓
[39]	✓	✓	✓	SS-IM-PP	×	✓	×	✓	✓	✓
[40]	✓	✓	✓	SS-IM-PP	✓	×	×	✓	×	✓
This paper	✓	✓	✓	SS-IM-PP	✓	×	✓	✓	✓	✓

Inventory management has also reportedly been analyzed for decades, with many models proposed to solve problems under some specific conditions, such as the demand type and the holding cost function. For example, the uncertain demand [41], [42], and nonlinear function [43] were emphasized in a recent publication, with a more advanced model also considering interval-valued production rate [44]. Furthermore, some applications were carried out with several types of goods, such as blood [45]. Irrespective of these conditions, inventory management integration with supplier selection and production planning problems is still limited. Based on Table 1, several related reports were observed regarding these production and selection issues. Therefore, this study aims to develop novel joint decision-making support for integrated supplier selection, production planning, and inventory management regarding the problem of discounted prices. These three supply-chain sub-problems were solved through the integrated flow of raw materials and products from the related supply parties. The proposed model was also tested through computational experiments using five horizon time instants.

This paper is structured as follows. The problem and the methodology are described in Section II. This section also contains the assumptions and the discount scheme adopted in the study. The main results are presented in Section III, containing the mathematical model proposed in this study. The mathematical model as the proposed decision-making support for the problem is formulated in this section. Finally, computational simulation results are presented in Section IV to illustrate and evaluate the proposed model for solving the given problem. Numerical experiment results and some managerial insights are also presented in this section.

II. MATERIALS AND METHOD

A. Problem Setting

Assuming a manufacturer is planning to buy R-types of raw materials from R-suppliers for T-future horizon time instants, a supply chain of raw materials and products is observed, as depicted in Fig. 1.

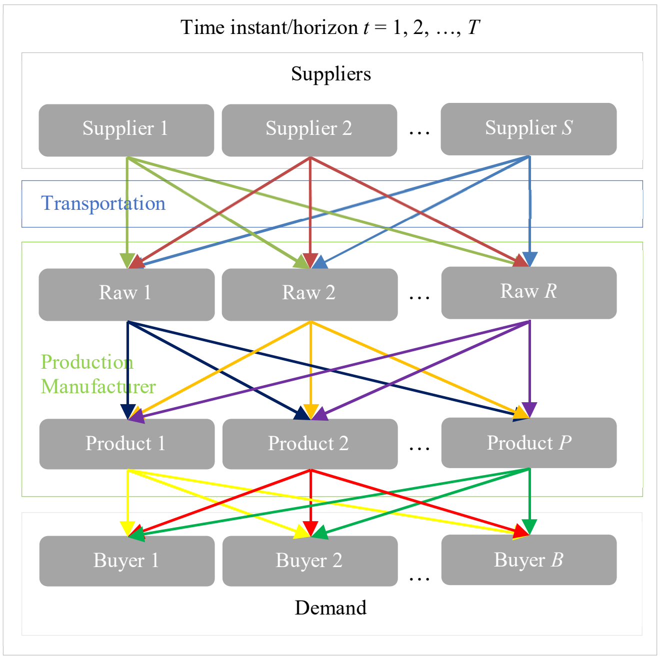

Fig. 1 The supply chain is composed of suppliers, production/assembly units, and buyers

This indicates that all suppliers, or only a few, are often selected to supply raw materials. In this case, a supplier is either eligible to supply all or only specific types of materials while being selected for some time instants. For instance, the ordered raw materials are commonly used to produce P product types at the present instant, with some goods also stored in the inventory for the next period. Based on the present time, the products are used/sold to satisfy buyers' demands, with some goods stored in the inventory for sale in the next period. In this case, problems often originate when determining the quantity of each raw material type, which should be ordered to each supplier and stored in the inventory at each time instant. These problems also originate when determining the quantity of each product type that should be produced and stored during each period, for a maximal profit of the overall supply-chain activity. In addition, the special specification of the problem emphasizes discounts on the raw material, transportation, inventory, and product prices.

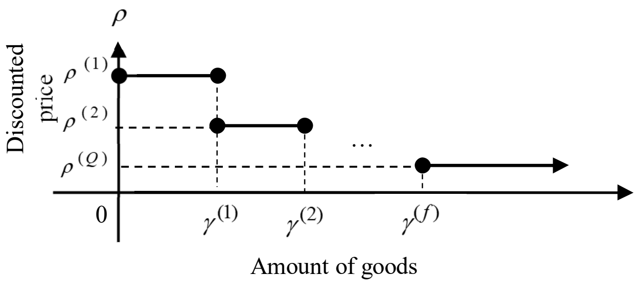

Fig.2 Discounted price scheme in a piecewise constant function

Subsequent details, including applicable assumptions for the problems' specifications, are explained as follows:

- The discounted prices emphasize piecewise constant functions, where costs are lower for different financial breakpoints, i.e., the more the goods, the cheaper the expenses (Fig. 2). This leads to the initial introduction of the price discount scheme, i.e., the more the goods, the cheaper the price. For example, at a discounted price ρ, when the amount of the purchased goods is $\leq \gamma^{(1)}$, then the unit cost = $\rho^{(1)}$. Meanwhile, when the amount of the purchased goods is $\geq \gamma^{(1)}$ and $\leq \gamma^{(2)}$, then the unit price = $\rho^{(2)}$. This is a common discount scheme used in most industries.
- Each supplier has its performance regarding capacities, discounts, as well as raw materials, transportation, rejection, and late delivery rates. This shows that a supplier is likely to have better prices and still be worse in other aspects, such as transportation costs. Therefore, the determination of the optimal decision becomes a non-trivial process.
- When arriving at the manufacturer, some raw materials are likely to be rejected due to damages and several unforeseen circumstances during the transportation processes. The rates for the damaged materials are also assumed to be known as a determined approximation by the decision-maker. In addition, penalty costs are applied for the damaged raw materials to fine the manufacturer's loss.
- The raw materials ordered at a specific time instant are assumed to be available to the manufacturer at that same period. However, shortage/late delivery is allowed during this process. The rates/percentages of the full ordered quantity represent this. These rates are assumably known as priori, with shortage/late delivery subsequently assumed to be obtained by the manufacturer at the next time instant. Penalty costs are also applied for these shortcomings to find the manufacturer's loss.
- A carrier is often contracted with discounted costs for transporting raw materials from suppliers to the manufacturer. This discount emphasizes the number of trucks used at each time instant.
- From a specific supplier, a one-truck transportation cost is incurred for transporting only raw materials to the manufacturer. In this case, mutual transportation from several suppliers is not considered.
- At a time instant, the available materials are used for production, and however, some are decisively stored in the inventory for subsequent productivity processes in the next period.
- Machines (or manufacturing steps) are commonly used for production processes in the production unit. The scheduling of these machines is mostly ignored in the model, with the whole production process assumably conducted in one time instant. A one-unit product is also likely carried out from several raw material types.
- Rejected/underqualified products are also observed and unable to be sold. These products were often assumed as non-valued materials with known rates/percentages.
- For a specific period, the goods produced are used to satisfy the demand from buyers. However, the quantity of these products is assumably more than the demand, with the excess materials stored in the inventory and used to satisfy the needs and wants at the next time instant.
- Product selling prices are assumed to have discounts, indicating that a buyer is likely to obtain goods at cheaper costs. However, these prices only emphasize the quantity of each product type purchased by the buyer.
- All measurements of raw materials and products are based on integer numbers.

B. *Methodology*

The following steps were used to solve the experimental issues, as summarized in Fig. 3.

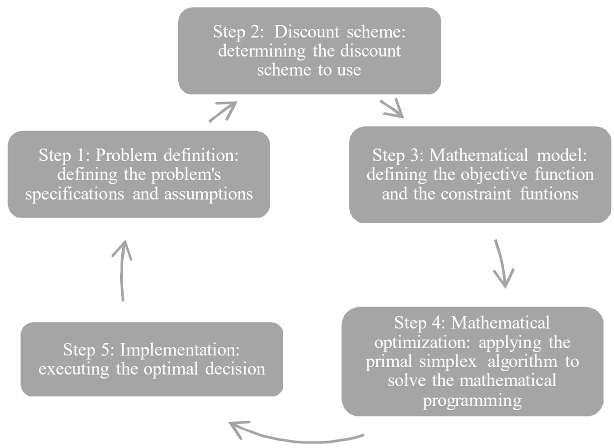

Fig. 3 Summary of the methodology

- Step 1: This emphasizes the problem encountered by the decision-maker, with the assumptions formulated based on the considered analytical conditions.
- Step 2: This used the piecewise-constant price schemes, as illustrated in Fig. 2. In this case, different schemes were either used for the whole problem or only for some discounted prices. However, the derived mathematical model was affected, with more complicated discount schemes producing more complex models.
- Step 3: In this process, each problematic component and profit are modelled as a mathematical and objective function, respectively. This objective case is observed as the function of the income and operational costs.

Meanwhile, for the constraint functions, these cases are either in equalities or inequalities, regarding the conditions to be met. The number of these functions also depends on the size of the problem, i.e., the rate of the suppliers, the raw material and product types, as well as the production machines.

- Step 4: The existing algorithm, the primal simplex is used in this process, for solving the derived mathematical optimization problem. Many other algorithms such as dual simplex and interior point method are also available in the literature, with selections depending on the decision maker's willingness and algorithm availability in the utilized software.
- Step 5: The derived optimal decision provided by the mathematical programming in the previous step is then implemented.

III. Results and Discussion

A. Notations

Consider the problem defined in the previous section, with T = time instants, S = suppliers, R = raw material types, M = production machines, P = product types, and B = buyers. The mathematical symbols used in the model are introduced in the following,

Indices:

- t : time instant, $t \in \{1,2,\ldots,T\}$,
- s : supplier, $s \in \{1,2,\ldots,S\}$,
- r : raw material, $r \in \{1,2,\ldots,R\}$,
- m : production machine, $m \in \{1,2,\ldots,M\}$,
- p : product, $p \in \{1,2,\ldots,P\}$,
- b : buyer, $b \in \{1,2,\ldots,B\}$,
- i,j,k,l,n : discount level.

Decision variables:

- X_{trs} : Amount of r purchased from s at t,
- Y_{tp} : Amount of p produced in the manufacturer at t,
- IR_{tr} : Amount of r stored in the warehouse at t,
- IP_{tp} : Amount of p stored in the warehouse at t.

Intermediate (semi-decision) variables:

- T_{ts} : number of trucks used for deliveries of r from s to the manufacturer at t,
- Z_s : Assignment variable to indicate whether s is selected to supply raw materials for the whole optimization time horizon$\{1,2,\ldots,T\}$.

Discounted parameters:

- $UP_{trs}^{(i)}$: Discounted price at i for the price of one unit r purchased from s at t,
- $PP_{tpb}^{(j)}$: Discounted price at j for the selling price of one unit p sold to b at t,
- $TC_{ts}^{(k)}$: Discounted price at k for one truck cost in transporting raw materials from s at t,
- $ICR_{tr}^{(l)}$: Discounted price at l for inventory cost of storing one unit r at t,
- $ICP_{tp}^{(n)}$: Discounted price at n for inventory cost of storing one unit p at t.

Parameters:

- O_s : Ordering cost for the whole optimization time horizon $\{1,2,\ldots,T\}$ to s,
- DR_{trs} : Damage/defect/rejection rate of r purchased from s at t,
- LR_{trs} : Late delivery/shortage rate of r purchased from s at t,
- MR_{mp} : The m hours needed to process one unit p at t,
- MM_{tm} : Maximum hour capacity of m to operate in one t,
- PC_{tp} : The cost needed for making one unit p at t,
- DP_{tp} : Under qualification/rejection/defect rate of produced p at t,
- DE_{tpb} : The demand of p from b at t,
- SC_{trs} : Maximum capacity of s in supplying r at t,
- TRC : Maximum capacity of one truck used in transporting r from s to the manufacturer, assuming that the trucks' capacities are equal and used for the whole optimization time horizon $\{1,2,\ldots,T\}$),
- PL_{trs} : Cost to penalize one unit late delivered r from s at t,
- PD_{trs} : Cost to penalize one unit rejected r from s at t,
- RR_{rp} : The required amount of r needed for producing one unit p,
- IR_{tr}^{max} : Maximum capacity of the warehouse to store r at t,
- IP_{tp}^{max} : Maximum capacity of the warehouse to store p at t.

B. Optimization Model

The adopted discounted price scheme was observed in the piecewise constant function (Fig. 2) and mathematically modeled. The relationship between the price and the product order volume is piecewise constant and represented by the following functions.

- Raw material price discount scheme,

$$UP_{trs} = \begin{cases} UP_{trs}^{(1)} & \text{if} \;\; X_{trs} \le X_{trs}^{(1)}, \\ UP_{trs}^{(2)} & \text{if} \;\; X_{trs}^{(1)} < X_{trs} \le X_{trs}^{(2)}, \\ \vdots & \\ UP_{trs}^{(I)} & \text{if} \;\; X_{trs} > X_{trs}^{(I-1)}; \end{cases} \tag{1}$$

- Product selling price discount scheme,

$$PP_{tpb} = \begin{cases} PP_{tpb}^{(1)} & \text{if} \;\; DE_{tpb} \le P_{tpb}^{(1)}, \\ PP_{tpb}^{(2)} & \text{if} \;\; DE_{tpb}^{(1)} < DE_{tpb} \le DE_{tpb}^{(2)}, \\ \vdots & \\ PP_{tpb}^{(J)} & \text{if} \;\; DE_{tpb} > DE_{tpb}^{(J-1)}; \end{cases} \tag{2}$$

- Transport cost discount scheme,

$$TC_{ts} = \begin{cases} TC_{ts}^{(1)} & \text{if} \;\; T_{ts} \le T_{ts}^{(1)}, \\ TC_{ts}^{(2)} & \text{if} \;\; T_{ts}^{(1)} < T_{ts} \le T_{ts}^{(2)}, \\ \vdots & \\ TC_{ts}^{(K)} & \text{if} \;\; T_{ts} > T_{ts}^{(K-1)}. \end{cases} \tag{3}$$

- Inventory cost discount scheme,

$$ICR_{tr} = \begin{cases} ICR_{tr}^{(1)} & \text{if} \quad IR_{tr} \le IR_{tr}^{(1)}, \\ ICR_{tr}^{(2)} & \text{if} \quad IR_{tr}^{(1)} < IR_{tr} \le IR_{tr}^{(2)}, \\ \vdots \\ ICR_{tr}^{(L)} & \text{if} \quad IR_{tr} > IR_{tr}^{(L-1)}. \end{cases} \tag{4}$$

$$ICP_{tp} = \begin{cases} ICP_{tp}^{(1)} & if \quad IP_{tp} \le IP_{tp}^{(1)}, \\ ICP_{tp}^{(2)} & if \; IP_{tp}^{(1)} < IP_{tp} \le IP_{tp}^{(2)}, \\ \vdots \\ ICP_{tp}^{(N)} & if \; IP_{tp} > IP_{tp}^{(N-1)}. \end{cases} \tag{5}$$

The discounted income (F_{0b}) from the buyer (b) is formulated as follows:

$$F_{tb}^{(0)} = \begin{cases} \sum_{p=1}^{P} [PP_{tp}^{(1)} \times DE_{tpb}] & \text{if} \quad DE_{tpb} \le DE_{tpb}^{(1)}, \\ \sum_{p=1}^{P} [PP_{tp}^{(2)} \times DE_{tpb}] & \text{if} \quad DE_{tpb}^{(1)} < DE_{tpb} \le DE_{tpb}^{(2)}, \\ \vdots \\ \sum_{p=1}^{P} [PP_{tp}^{(J)} \times DE_{tpb}] & \text{if} \quad DE_{tpb} > DE_{tpb}^{(J-1)}. \end{cases}$$

Based on these models, operational costs and their corresponding mathematical expressions were considered as follows:

- Ordering cost for a selected supplier to supply raw materials,

$$F^{(1)} = \sum_{s=1}^{S} [OC_s \times Z_s]. \tag{6}$$

- Purchasing costs based on the price discount scheme,

$$F_t^{(2)} = \begin{cases} \sum_{r=1}^{R} \sum_{s=1}^{S} [UP_{trs}^{(1)} \times X_{trs}] & \text{if} \quad X_{trs} \le X_{trs}^{(1)}, \\ \sum_{r=1}^{R} \sum_{s=1}^{S} [UP_{trs}^{(2)} \times X_{trs}] & \text{if} \quad X_{trs}^{(1)} < X_{trs} \le X_{trs}^{(2)}, \\ \vdots \\ \sum_{r=1}^{R} \sum_{s=1}^{S} [UP_{trs}^{(I)} \times X_{trs}] & \text{if} \quad X_{trs} > X_{trs}^{(I-1)}. \end{cases} \tag{7}$$

- Transportation costs regarding the price discount scheme,

$$F_t^{(3)} = \begin{cases} \sum_{s=1}^{S} [TC_{ts}^{(1)} \times T_{ts}] & \text{if} \; T_{ts} \le T_{ts}^{(1)}, \\ \sum_{s=1}^{S} [TC_{ts}^{(2)} \times T_{ts}] & \text{if} \; T_{ts}^{(1)} < T_{ts} \le T_{ts}^{(2)}, \\ \vdots \\ \sum_{s=1}^{S} [TC_{ts}^{(J)} \times T_{ts}] & \text{if} \; T_{ts} > T_{ts}^{(J-1)}. \end{cases} \tag{8}$$

- Total inventory cost concerning the price discount scheme,

$$F_t^{(4)} = \begin{cases} \sum_{r=1}^{R} [ICR_{tr}^{(1)} \times IR_{tr}] & \text{if} \; IR_{tr} \le IR_{tr}^{(1)}, \\ \sum_{r=1}^{R} [ICR_{tr}^{(2)} \times IR_{tr}] & \text{if} \; IR_{tr}^{(1)} < IR_{tr} \le IR_{tr}^{(2)}, \\ \vdots \\ \sum_{r=1}^{R} [ICR_{tr}^{(L)} \times IR_{tr}] & \text{if} \; IR_{tr} > IR_{tr}^{(L-1)}. \end{cases} \tag{9}$$

$$F_t^{(5)} = \begin{cases} \sum_{p=1}^{P} [ICP_{tp}^{(1)} \times IP_{tpp}] & \text{if} \; IP_{tp} \le IP_{tp}^{(1)}, \\ \sum_{p=1}^{P} [ICP_{tp}^{(2)} \times IP_{tp}] & \text{if} \; IP_{tp}^{(1)} < IP_{tp} \le IP_{tp}^{(2)}, \\ \vdots \\ \sum_{p=1}^{P} [ICP_{tp}^{(N)} \times IP_{tp}] & \text{if} \; IP_{tp} > IP_{tp}^{(N-1)}. \end{cases} \tag{10}$$

- The total penalty cost for late delivered raw materials,

$$F_t^{(6)} = \sum_{r=1}^{R} \sum_{s=1}^{S} [PL_{trs} \times LR_{trs} \times X_{trs}]. \tag{11}$$

- The total penalty cost for defected materials,

$$F_t^{(7)} = \sum_{r=1}^{R} \sum_{s=1}^{S} [PD_{trs} \cdot DR_{trs} \cdot X_{trs}]. \tag{12}$$

- The total production cost,

$$F_t^{(8)} = \sum_{p=1}^{P} [PC_{tp} \times Y_{tp}]. \tag{13}$$

Based on the considered problem definition and the assumptions, the constraint functions are modeled as follows:

- Raw material demand satisfaction: For each material type, the available amount at each instant has to satisfy the demand for production. This is formulated as follows:

  The amount of the raw material (r) from the previous and present time instant (t) + the late delivered r from the existing and present t - the rejected r at the present t - the rate of r decided to be stored in the inventory.

  This formulation has to be no less than the number of raw materials needed to manufacture products at the present instant. It is also modeled as the following inequality:

$$IR_{(t-1)r} + \sum_{s=1}^{S} X_{trs} + \sum_{s=1}^{S} [LR_{(t-1)rs} \times X_{(t-1)rs}] - \sum_{s=1}^{S} [LR_{trs} \times X_{trs}] - \sum_{s=1}^{S} [DR_{trs} \times X_{trs}] - IR_{tr} \ge \sum_{p=1}^{P} [RR_{rp} \times Y_{tp}] \tag{14}$$

  At the time instant, $t = 1$, the values at the previous t indicated the initial values. This value is likely set as zero when no initial inventory or ordered raw materials are observed.

- Product demand satisfaction: The formulation of this function has to be no less than the amount demanded by all buyers for each product type. This is modelled as follows:

$$IP_{(t-1)p} + Y_{tp} - [DP_{tp} \times Y_{tp}] - IP_{tp} \ge \sum_{b=1}^{B} DE_{tpb}; \tag{14}$$

- Selected supplier(s) indication: When a supplier is selected to supply some raw materials at any time instant, the order cost is highly needed. Since it is a one-time cost for the whole-time horizon, the indicator, Z_s, is then used to assign whether the supplier (s) is selected or not. This is designed with the following function:

$$Z_s = \begin{cases} 1 & \text{if} \;\; \sum_{t=1}^{T} \sum_{r=1}^{R} X_{trs} > 0, \\ 0 & \text{otherwise}; \end{cases} \tag{15}$$

- Calculation of trucks used for transportation: When deciding the number of trucks used in transporting r from s to the manufacturer, the following inequality needs to be satisfied for each t and s, as follows:

$$\left\lceil \frac{\sum_{r=1}^{R} X_{trs}}{TRC} \right\rceil \le T_{ts}; \tag{16}$$

  This indicates that at each time instant and supplier, the total amount of all raw materials divided by the truck capacity (rounded up by the ceiling function $\lceil . \rceil$) is less than the number of vehicles used for transportation.

- Production machines capacity limitation: For all machines, the total working hour to manufacture all products at each time instant did not exceed its maximum operating period. This is modeled as follows:

$$\sum_{p=1}^{P}[MR_{mp} \times Y_{tp}] \leq MM_{tm}; \quad (17)$$

- Capacity limits of suppliers and warehouses: For each time instant, all raw materials, suppliers, products, as well as ordering and storing values should not exceed the limits. These are modeled as the following inequalities:

$$X_{trs} \leq SC_{trs}; \quad (18)$$

$$IR_{tr} \leq IR_{tr}^{max} \quad (19)$$

$$IP_{tp} \leq IP_{tp}^{max} \quad (20)$$

- Nonnegativity and integer: All decision variables are nonnegative and integer when applicable. This is expressed as follows:

$$X_{trs}, T_{ts}, Y_{tp} \geq 0 \text{ and integer.} \quad (21)$$

Combining all these cost and constraint functions, the following mathematical optimization problem is obtained,

$$max\, Z = \sum_{t=1}^{T}\sum_{b=1}^{B}[F_{0b}] - F^{(1)} - \sum_{t=1}^{T}\sum_{c=2}^{8}[F_t^{(c)}] \quad (22)$$

subject to as follows:

$$\forall t, r: \quad IR_{(t-1)r} + \sum_{s=1}^{S} X_{trs} + \sum_{s=1}^{S}[LR_{(t-1)rs} \times X_{(t-1)rs}] - \sum_{s=1}^{S}[LR_{trs} \times X_{trs}] - \sum_{s=1}^{S}[DR_{trs} \times X_{trs}] - IR_{tr} \geq \sum_{p=1}^{P}[RR_{rp} \times Y_{tp}]. \quad (24)$$

$$\forall t, p: \quad IP_{(t-1)p} + Y_{tp} - [DP_{tp} \times Y_{tp}] - IP_{tp} \geq \sum_{b=1}^{B} DE_{tpb}; \quad (23)$$

$$Z_s = \begin{cases} 1 & \text{if } \sum_{t=1}^{T}\sum_{r=1}^{R} X_{trs} > 0, \\ 0 & \text{otherwise;} \end{cases} \quad (24)$$

$$\forall t, s: \quad \left\lceil \frac{\sum_{r=1}^{R} X_{trs}}{TRC} \right\rceil \leq T_{ts}; \quad (25)$$

$$\forall t, m: \quad \sum_{p=1}^{P}[MR_{mp} \times Y_{tp}] \leq MM_{tm}; \quad (26)$$

$$\forall t, r, s: \quad X_{trs} \leq SC_{trs}; \quad (27)$$

$$\forall t, r: \quad IR_{tr} \leq IR_{tr}^{max} \quad (28)$$

$$\forall t, p: \quad IP_{tp} \leq IP_{tp}^{max} \quad (29)$$

$$\forall t, r, s, p: \quad X_{trs}, T_{ts}, Y_{tp} \geq 0 \text{ and integer.} \quad (30)$$

This is a linear programming model with a piecewise objective function. For the existence of the optimal solution, an issue was not found as long as the feasible set is compact and not empty, i.e., closed and bounded.

C. Computational Simulation Results

Computational simulations were performed to highlight the patterns by which the proposed model was implemented. These were carried out in a small-scale laboratory, using a computer with common specifications to verify and evaluate the proposed model. All data used for the experiments were also randomly generated.

Using two machines, M1 and M2, consider a manufacturer is optimizing its production planning for five future time instants, to manufacture four products, P1, P2, P3, and P4, from 4 raw materials, R1, R2, R3, and R4. In this case, four suppliers, S1, S2, S3, and S4, were also assumed to supply these materials. This model (23) emphasized the determination of the optimal decision to maximize profit. Furthermore, prices, including raw material, product, transportation, and inventory costs, were discounted with the piecewise schemes of the three discount levels (or price break points) denoted by DL1, DL2, and DL3. These discounted prices were subsequently supporting the following piecewise functions,

$$\forall t: \quad UP_{trs} = \begin{cases} UP_{trs}^{(1)} & \text{if } X_{trs} \leq 100, \\ UP_{trs}^{(2)} & \text{if } 100 < X_{trs} \leq 200, \\ UP_{trs}^{(3)} & \text{if } X_{trs} > 200; \end{cases} \quad (31)$$

$$\forall t: \quad UP_{trs} = \begin{cases} UP_{trs}^{(1)} & \text{if } X_{trs} \leq 100, \\ UP_{trs}^{(2)} & \text{if } 100 < X_{trs} \leq 200, \\ UP_{trs}^{(3)} & \text{if } X_{trs} > 200; \end{cases} \quad (32)$$

$$\forall t: \quad PP_{tpb} = \begin{cases} PP_{tpb}^{(1)} & \text{if } DE_{tpb} \leq 10, \\ PP_{tpb}^{(2)} & \text{if } 10 < DE_{tpb} \leq 15, \\ PP_{tpb}^{(3)} & \text{if } DE_{tpb} > 15; \end{cases} \quad (33)$$

$$\forall t: \quad TC_{ts} = \begin{cases} TC_{ts}^{(1)} & \text{if } T_{ts} \leq 1, \\ TC_{ts}^{(2)} & \text{if } 2 < T_{ts} \leq 5, \\ TC_{ts}^{(3)} & \text{if } T_{ts} > 5; \end{cases} \quad (34)$$

$$\forall t: \quad ICR_{tr} = \begin{cases} ICR_{tr}^{(1)} & \text{if } IR_{tr} \leq 10, \\ ICR_{tr}^{(2)} & \text{if } 10 < IR_{tr} \leq 20, \\ ICR_{tr}^{(3)} & \text{if } IR_{tr} > 20; \end{cases} \quad (35)$$

$$\forall t: \quad ICP_{tp} = \begin{cases} ICP_{tp}^{(1)} & \text{if } IP_{tp} \leq 10, \\ ICP_{tp}^{(2)} & \text{if } 10 < IP_{tp} \leq 20, \\ ICP_{tp}^{(3)} & \text{if } IP_{tp} > 20; \end{cases} \quad (36)$$

where, $i, j, k, l, n = 1,2,3$ and $t = 1,2, \ldots, 5$, $UP_{trs}^{(i)}$, $PP_{tpb}^{(j)}$, $TC_{ts}^{(k)}$, $ICR_{tr}^{(l)}$, and $ICP_{tp}^{(n)}$ are shown in Tables 2 to 6. In addition, the values for the other parameters are shown in Tables 7 to 12.

TABLE II
DISCOUNTED RAW MATERIAL PRICES $UP_{trs}^{(i)}$

Time Instant	Supplier	*Discount level*	Raw material			
			R1	***R2***	***R3***	***R4***
all	S1	DL1	50	20	35	55
		DL2	45	18	30	50
		DL3	45	17	30	50
	S2	DL1	50	22	37	55
		DL2	50	21	35	55
		DL3	45	21	35	52
	S3	DL1	50	20	30	55
		DL2	48	20	30	52
		DL3	45	20	30	50
	S4	DL1	50	20	32	57
		DL2	45	19	32	55
		DL3	45	18	30	50

TABLE III

DISCOUNTED PRODUCT SELLING PRICES $PP_{tpb}^{(j)}$

Time instant	Product	Buyer		
		All		
		DL1	***DL2***	***DL3***
all	P1	240	200	200
	P2	400	350	300
	P3	400	400	350
	P4	400	400	350

TABLE IV

DISCOUNTED TRANSPORT COSTS DATA $TC_s^{(j)}$

Time instant	Supplier	Discount Level		
		DL1	***DL2***	***DL3***
all	S1	120	110	105
	S2	120	115	110
	S3	125	110	110
	S4	120	120	110

TABLE V

DISCOUNTED RAW MATERIAL INVENTORY COSTS $ICR_{tr}^{(l)}$

Time instant	Supplier	Discount Level		
		DL1	***DL2***	***DL3***
all	R1	5.00	4.00	3.00
	R2	2.00	2.00	1.00
	R3	1.50	1.25	1.00
	R4	2.00	1.50	1.00

TABLE VI

DISCOUNTED PRODUCT INVENTORY COSTS $ICP_{tp}^{(n)}$

Time instant	Supplier	Discount Level		
		DL1	***DL2***	***DL3***
all	P1	2.00	1.00	1.00
	P2	2.00	2.00	1.00
	P3	1.50	1.00	1.00
	P4	2.00	1.50	1.00

TABLE VII

THE RATES OF DEFECTED RAW MATERIALS DR_{trs}

Time instant	Supplier	Raw material type		
		R1	***R2***	***R3***
all	S1	0.044	0.030	0.037
	S2	0.019	0.043	0.030
	S3	0.020	0.038	0.050
	S4	0.014	0.034	0.016

TABLE VIII

THE RATES OF LATE DELIVERED RAW MATERIALS LR_{trs}

Time instant	Supplier	Raw material type		
		R1	***R2***	***R3***
all	S1	0.028	0.035	0.033
	S2	0.028	0.035	0.031
	S3	0.035	0.049	0.012
	S4	0.027	0.021	0.014

TABLE IX

SUPPLIER'S MAXIMUM CAPACITY SC_{trs}

Time instant	Raw material	Supplier				
		S1	***S2***	***S3***	***S4***	***S5***
all	R1	2500	2200	2000	2000	2500
	R2	1500	1800	1500	1200	1500
	R3	2000	1500	2500	2500	2000
	R4	2000	1500	2500	2500	2000

TABLE X

REQUIRED MACHINE HOUR FOR PRODUCTION MR_{mp}

Machine	Product			
	P1	***P2***	***P3***	***P4***
M1	1	1	2	1
M2	2	1	1	2

TABLE XI

PRODUCT DEMAND FROM BUYERS DE_{tpb}

Time instant	Product	Buyer				
		B1	***B2***	***B3***	***B4***	***B5***
1	P1	30	30	30	20	30
	P2	5	30	30	10	0
	P3	20	20	10	10	20
	P4	20	0	10	10	10
2	P1	30	30	10	20	20
	P2	20	10	10	10	10
	P3	10	10	20	30	20
	P4	20	0	30	30	10
3	P1	10	30	20	20	20
	P2	5	30	20	10	30
	P3	10	10	30	10	30
	P4	0	30	10	30	20
4	P1	30	10	30	30	10
	P2	10	10	20	20	10
	P3	0	10	10	30	10
	P4	10	30	20	20	20
5	P1	10	20	30	20	30
	P2	30	10	20	0	20
	P3	0	10	30	20	20
	P4	10	20	10	0	10

Using the defined parameters, the optimization problem was solved in the LINGO 19.0 software. The algorithm used in the simulations was the linear programming primal simplex. To determine the integer solutions, the branch-and-bound algorithm was employed. Based on the supplier selection, raw material inventory, production planning, and product inventory, the optimal decisions derived from the optimization are shown in Figs. 4, 5, and 6, respectively.

According to these decisions, only S1 and S2 were selected to supply raw materials. Only S2 supplied R1 at the first, second, and fourth instant. Fig. 5 shows the inventory level of raw materials for each period, indicating that only R2 was stored at the first time instant. Meanwhile, no raw material was needed for storage during the second, third, and fifth periods. This indicated that purchasing raw materials at these time instants was more beneficial than storage. During the fifth period, the decision not to store materials was due to no production at the future time instants, confirming that the best decision was to utilize all the available raw materials for the manufacture of products.

TABLE XII

OTHER PARAMETERS

time instant	Parameter	Supplier/raw/product/ machine		
		S1/R1/ P1/M1	***S2/R2/ P2/M2***	***S3/R 3/P3***
all	Order cost (OC_{ts})	50	50	40
	Defected raw material penalty cost (PD_{trs})	1	1.5	2
	Late delivered raw material penalty cost (PL_{trs})	2	1	2
	Production cost (PC_{tp})	10	5	12
	Product defect rate (DP_{tp})	0.014	0.012	0.019
	Warehouse capacity for storing raw materials (IR_{tr}^{max}	100	50	100
	Warehouse capacity for storing products(IP_{tp}^{max}	50	45	50
	Maximum machine working hour (MM_{tm})	2500	2000	NA

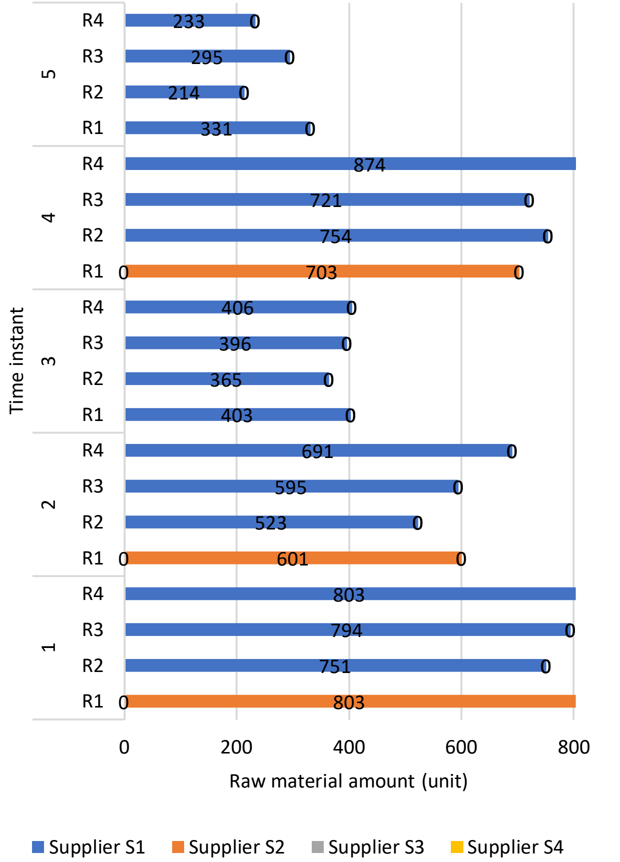

Fig. 4 The optimal decision regarding the amount of each raw material type that should be ordered from each supplier

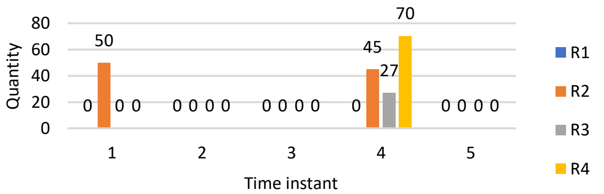

Fig. 5 The optimal decision regarding the inventory of raw materials

Based on Fig. 6, the optimal decisions regarding production planning and the inventory of products are shown in one chart. This indicated the qualified products, i.e., the quantity of the produced goods minus the rejected types. It also exhibited the on-hand products, showing the quantity of the available goods to be sold, including those from the inventory. From these results, the on-hand products satisfied the demands at each time instant, with the cost and income at 408928.8 and 524000.0, leading to a profit of 115071.2, respectively.

D. Managerial Insights

Regarding the derived mathematical model and the simulation results, the following managerial insights were obtained:

The proposed model is eligible for use in handling different types of raw materials/products within any sector, although the assumptions should be highly considered. Moreover, the model is likely to be slightly modified, regarding the assumptions employed by the decision-maker. For instance, when a budget limit is observed for the total operational cost, an additional constraint should be added to incorporate this situation.

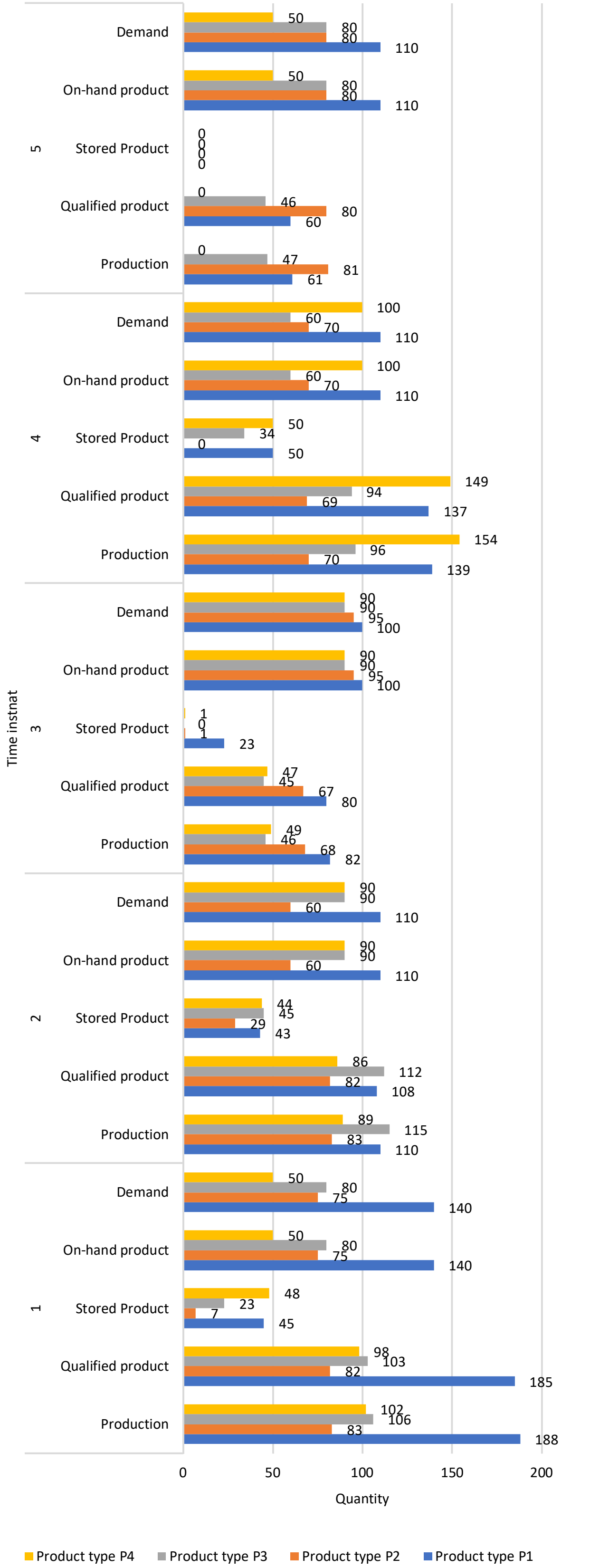

Fig. 6 The optimal decision regarding the production volume, the actual qualified product and its corresponding demand

The adopted simulations were academic examples, due to being small-scale problems based on the optimization size. This was because all computations were carried out within minutes. In this case, caution should be observed with large-scale problems, due to the need for longer computational time. Therefore, a personal computer is likely insufficient to perform the computation.

When the set of feasible solutions is empty, subsequent analysis is needed to solve the optimization problem. In this situation, the problem needs to be relaxed for the determination of solutions. For example, when all suppliers' total maximum capacity is insufficient to satisfy the specific requirements, the problem should be relaxed by deciding not to completely meet the demand.

The limitation of the available production machine hours is also the cause of insufficient product-demand satisfaction due to maintenance and other related factors. This is likely solved by modifying the constraint function of the satisfaction process through the replacement of $\geq$ with $\leq$.

IV. Conclusion

An integrated decision-making support model was proposed to solve the problems of supplier selection and production planning, regarding discounted prices. This model emphasized an objective optimization structure, where the discounted prices were considered as piecewise constant functions. Based on the simulations, the proposed model successfully determined the optimal decision regarding raw material procurement and production, providing a maximal profit.

Acknowledgment

The authors are grateful to RPI UNDIP 2022 project for financially supporting this study under contract no. 569-112/UN7.D.2/PP/VII/2022.

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