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Algorithm

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Abstract— University timetabling construction is a complicated task that universities worldwide encounter. This study developed a hybrid approach to produce a timetable solution for the university examination timetabling problem. Black Hole Algorithm (BHA), a population-based approach that mimics the black hole phenomenon, has recently been introduced in the literature and successfully applied in addressing various optimization problems. Although its effectiveness has been proven, there still exists inefficiency regarding the exploitation ability where BHA is poor in fine-tuning search region in reaching for good quality of the solution. Hence, a hybrid framework for university examination timetabling problem that is based on BHA and Hill Climbing local search is proposed (hybrid BHA). This hybridization aims to improve the exploitation ability of BHA in fine-tuning the promising search regions and convergence speed of the search process. A real-world university examination benchmark dataset has been used to evaluate the performance of hybrid BHA. The computational results demonstrate that hybrid BHA can generate competitive results and record the best results for three instances compared to the reference approaches and current best-known recorded in the literature. Besides, findings from the Friedman tests show that the hybrid BHA ranked second and third in comparison with hybrid and meta-heuristic approaches (a total of 27 approaches) reported in the literature, respectively.

Keywords- Black hole algorithm; university examination timetabling; meta-heuristic; optimization.

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I. INTRODUCTION

Examination timetabling has been a challenging combinatorial problem that has attracted researchers' attention over three decades. University examination timetabling is a tedious administrative task that needs to be done in all academic institutions every semester [1], [2] where a set of exams must be allocated into permitted time slots and rooms while fulfilling a set of constraints (hard and soft) [3]. It should be noted that the constraints considered differ greatly across different institutions, and violation of hard constraints is prohibited. Satisfaction with soft constraints is not absolute and is used to assess the quality of a timetable solution. It should be noted that a higher satisfaction with soft constraints will result in a better quality of timetable solution.

In computing terms, examination timetabling can be assumed to be a NP-hard optimization problem consisting of a huge search space with multiple locally optimal solutions [1]-[4]. Hence, an approach with strong search characteristics is needed to generate a good timetable solution [1]. Survey papers reported by precious studies show that various automated approaches have been proposed for university examination timetabling problems [2]-[5]. At the early stage, heuristic approaches [3], [6], [7] are used. However, the performance is relatively poor due to the generated timetable solution with a low degree of soft constraints satisfaction. Meta-heuristic approaches have been studied and applied in solving university timetabling problems. Many research works reported have demonstrated that meta-heuristic approaches are more sophisticated and capable of producing timetable solutions with good satisfaction on soft constraints compared to heuristics [5]. This is due to meta-heuristic approaches with search abilities (exploration and exploitation) that can explore and exploit the solution search regions to reach an optimal solution [8]. Exploration refers to the searchability that can explore unvisited search regions in the

search space [8]. Exploitation refers to the ability to explore a search region in reaching areas with good quality solutions [8].

Single solution-based approaches with good exploitation ability. These include tabu search [9], [10], simulated annealing [11], late acceptance hill-climbing [12], and great deluge algorithm [13]–[14]. Population-based approaches are with good exploration ability. For examples, include the firefly algorithm [15], honey bee mating algorithm [16], harmony search [17], [18], and artificial bee algorithm [19]– [21].

Other than using meta-heuristic approaches alone, hybridization of several approaches has been proposed to overcome the limitations of using meta-heuristic approaches independently. Single solution-based approaches are good for fine-tuning a search region for an optimal local solution. For population-based approaches, their strength is the ability to search for solutions at different search regions (unvisited or promising) simultaneously. Hence, hybridizing single and population-based approaches can benefit both approaches in exploring and fine-tuning search regions in the search space. The research found that the hybrid approach usually performs better as it combines the advantage of different approaches into one in addressing a problem [19], [20], [53]. Burke et al. [22] proposed a hybrid method that combines a genetic algorithm and variable neighborhood search for the examination timetabling problem. The chromosome in the genetic algorithm is used to present a list of neighborhood structures (component from variable neighborhood search) rather than a solution. With a list of different neighborhood structures, it would improve the exploitation ability of the conventional genetic algorithm in exploring solutions in a particular search region. Literature-beating results were produced in certain instances. Hybridization of an electromagnetic-like approach with the great deluge algorithm is proposed by Turabieh and Abdullah [14] for university timetabling problems. In this approach, the electromagnetic-like mechanism is used to calculate the forces between the solutions, which is used to calculate the estimated quality and decay rate (parameters) of the great deluge algorithm. Results were produced to show that the hybrid approach performed well in exploring the search region. The same researchers introduced another hybrid approach, combining fish swarm optimization, great deluge algorithm, and nelder-mead simplex search for the same timetabling problem [13].

The combination of fish swarm optimization and neldermead simplex search is to calculate decay rate, a parameter for great deluge algorithm in exploiting a particular search region. Promising results were attained. Abed and Tang [23] hybridized genetic algorithm with a record-to-record travel algorithm for university examination timetabling problem. Experimental results showed that the record-to-record travel algorithm empowered the exploitation ability of genetic algorithms by generating better outcomes than the genetic algorithm alone. Alzaqebah and Abdullah [24] hybridized bee colony algorithm with late acceptance hill-climbing local search for university examination timetabling problem. Competitive results were generated, showing the effectiveness of late acceptance hill-climbing local search in improving the exploitation of bee colony algorithm. Aldeeb et al. [25] presented a hybrid method based on intelligent water drops and local search algorithms. The latter is used to enhance the exploitation ability of the solution searching process. A university timetabling benchmark dataset is selected to examine the performance of the hybrid approach. The results show that the hybrid approach generated good results in certain instances compared to other approaches published in the previous studies. Fong et al. [20] investigated university timetabling problems. A hybridization between artificial bee colony algorithm, imperialist competitive algorithm, and great deluge algorithm is introduced. The hybridization aims to achieve a balance between exploration and exploitation of search abilities. Competitive outcomes were achieved for the benchmark dataset studied. Leite et al. [26] hybridized cellular memetic algorithm with threshold acceptance local search for university examination timetabling problem. The hybridization aims to empower the exploitation of cellular memetic algorithms. Literaturebeating results were produced for a few instances of the problem studied. Jahwar et al. [27] proposed a hybrid approach based on a combination of genetic algorithm and swarm optimization in solving university particle examination timetabling problems. Experimental results show that the hybrid approach performed better than the conventional approaches. However, the proposed approach suffers from poor exploitation ability where the search region of the promising search region is not properly explored. Alefragis et al. [28] proposed a multi-meta-heuristic variable neighborhood search approach where simulated annealing, late acceptance hill climbing, and flex deluge algorithms are applied after a solution is generated by variable neighborhood search. Promising results were produced on the university examination timetabling benchmark dataset.

The BHA proposed by Hatamlou [29] mimics the black hole phenomenon. The BHA begins with an initial population, and the best candidate is selected as a black hole. During the iteration of BHA, the black hole will pull the stars (the rest of the candidates) towards its position. The black hole will swallow a star if it is too close to the black hole. In this situation, a new star (new candidate) will be produced randomly and placed in the search space. The BHA algorithm's strengths are its ability to explore (generating a new solution randomly in the search region to escape from local optima through the phenomena where a black hole swallows a star) and exploiting (a star pulling towards a black hole) the problem search region. However, it is still having difficulty in reaching for optimal solutions due to the imbalance of exploration and exploitation abilities where the exploitation is slow (movement towards black hole) and the new solution that generated randomly to replace the old solution (swallowed by a black hole) has decreased further the exploitation, resulting in slow convergence.

Motivated by the successfulness of many studies that are based on the idea of hybridization in tackling university examination timetabling problems and the inefficiency of BHA mentioned above, this study proposed a hybrid approach based on BHA and Hill Climbing (HC) local search. The use of HC aims to enhance the exploitation of BHA, resulting in a hybrid approach with good balance of exploration and exploitation abilities that ensure the promising search regions are fine-tuned in reaching good quality solutions. Experimental results show that search abilities improved when the modification stated above is applied.

The rest of this paper is presented as Section II provides a description of the university examination timetabling problem studied in this work. Sections III and IV give the general idea of the basic and the proposed hybrid algorithms, respectively. Section V discussed the experimental results obtained, and Section VI summarized the work done in this study.

II. MATERIAL AND METHOD

The university examination timetabling problem deals with a set of exams, time slots, rooms, and a set of constraints that must be satisfied. Two types of constraints are considered in university timetabling problems: hard and soft. Satisfaction with hard constraints is compulsory for a timetable solution and is known as a feasible timetable. For soft constraint, satisfaction is required, but it is not essential. The quality of a timetable solution is measured based on the degree of violation of the soft constraints. In other words, the violation of soft constraints should be minimized. It should be noted that a penalty will be applied if there is a violation of soft constraints, and the penalty may vary for each violation.

Carter's un-capacitated examination timetabling dataset [30] is selected to assess the performance of the hybrid approach proposed in this study. It is a well-established un-capacitated examination timetabling dataset that consists of real-world instances contributed by several institutions. Besides, many research researchers have adopted this dataset in evaluating the performances of their proposed approaches [11], [24]–[27], [31]–[36], [51], [52]. This dataset's characteristics, input, and the constraints considered are presented in Tables I, II, and III, respectively.

 TABLE I

 CARTER'S UN-CAPACITATED EXAMINATION DATASET

Instance	Timeslot	Exam	Student	Density
car92	32	543	18,419	0.14
car91	35	682	16,925	0.13
ear83	24	190	1125	0.27
hec92	18	81	2823	0.42
kfu93	20	461	5349	0.06
lse91	18	381	2726	0.06
sta83	13	139	611	0.14
tre92	23	261	4360	0.18
uta92	35	622	21,267	0.13
ute92	10	184	2750	0.08
yor83	21	181	941	0.29
				

TABLE II INPUT FOR CARTER'S DATASET

Symbol	Description
N	number of exams
M	number of students
Р	a predefined timeslot
С	Conflict matrix, $c=(c_{ij})_{N\times N}$ where each element in the symmetrical matrix is the number of students that sit
	for both exams i and j , where $i, j \in 1,, N$.
ti	t_i is the timeslots that scheduled exam i ($j \in 1,,N$) within the set of predefined timeslots ($1 \le t_i \le P$)

TABLE III HARD AND SOFT CONSTRAINTS FOR CARTER'S DATASET

Constraints	Description
Hard	No students are requested to attend two or more
constraint	examinations at one time
Soft	A large time gap between two conflicting
constraint	examinations to allow students to have enough
	time for the revision process on registered
	examinations.

The penalty cost function (1) is formulated to measure the quality of a feasible timetable solution (violation of soft constraint).

$$minimize \frac{\sum_{i=1}^{N-1} F_i(i)}{M} \tag{1}$$

Where

$$F(i) = \sum_{j=i+1}^{N} C_{ij} \cdot proximity(t_i, t_j)$$
(2)

and

$$oximity(t_i, t_i) = \begin{cases} 2^5/2^{|t_i - t_j|} & \text{if } 1 \le |t_i - t_j| \le 5\\ 0 & \text{otherwise} \end{cases}$$
(3)

subject to

pr

$$\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} c_{ij} \lambda(t_i, t_j) = 0$$
(4)

where

$$A(t_i, t_j) = \begin{cases} 1 & if \ t_i = t_j \\ 0 & otherwise \end{cases}$$
(5)

Equation (2) represents the formulation to calculate violation on the soft constraint, which is the multiplication of the proximity value between two conflict examinations (calculate using (3)) with the sum of the number of students registered in the conflict examinations. A proximity value of 16 (calculated using (3)) will be applied if a student is required to attend two consecutive exams. A proximity value of 8 will be counted if there is one free gap between two exams registered by a student. A value of 4 will be counted if there are two free time slots between two conflicting exams.

Equations (4) and (5) represent formulations to measure satisfaction on the hard constraint (as presented in Table III). The encoding scheme that is used to represent a candidate solution for the university examination timetabling problem is presented in Fig. 1.

	t_1	t ₂	t3	t4		t_t
exam	e1	e ₁₉	e ₁₀	e ₂₈	e ₉	e 7
exam	e ₆	e ₁₅	e ₁₃	e ₂₂	e ₁₇	e_1
exam	e ₂₆	e ₂₅	e_6	e ₃₁	e ₂₇	
exam	e ₁₈			e_{21}		
exam						

Fig. 1 Encoding scheme for university examination timetabling

A. Proposed Hybrid BHA For University Examination Timetabling Problem

This section discusses the basic concept of BHA, its application of BHA in university examination timetabling problem, hill climbing local search, and the neighborhood structures used.

B. Black Hole Algorithm (BHA)

The BHA is a population-based metaheuristic approach introduced by Hatamlou [29] that emulates the black hole phenomenon. Before the solution searching process, a population of solutions is generated and distributed randomly in the solution search space. This is then followed by a repeating cycle of evolving the population towards the optimal solution through certain solution searching mechanisms.

In BHA, the population is generated randomly in the search space and referred to as stars. Next, the population's fitness will be measured using a fitness function of a given problem. The candidate solution with the best quality will be selected as a black hole, and the remaining candidate solutions in the population will be the stars. A black hole has a very strong gravity, and it will absorb the stars surrounding it. At each iteration, the black hole will absorb the surrounding stars and all stars will move towards the black hole by using (6).

$$x_i(t+1) = x_i(t) + rand \times (x_{BH}, x_i(t)) \quad i = 1, 2, \dots, N$$
 (6)

where,

- $x_i(t)$ is the position of *i*th star at iteration *t*;
- $x_i(t+1)$ is the position of *i*th star at iteration t+1;
- x_{BH} is the position of black hole;
- *rand* is a random value in the range of [0,1];
- N is the number of stars (solutions) in the population.

During the movement towards the black hole, a star may reach a position with better fitness than the black hole. In this situation, the star will become the new black hole and the old black hole will turn into a star. Next, the rest of the stars will continue to move towards the location of the new black hole. Besides, the black hole will swallow any of the stars that cross the event horizon of the black hole. Every time the black hole sucks a star, a new star will be generated randomly and placed in the search space. This replacement is to ensure the number of solutions remains unchanged.

The radius, R for the event horizon of the BHA is measured using (7).

$$R = \frac{f_{BH}}{\sum_{i=1}^{N} f_i} \tag{7}$$

where,

- f_{BH} is the fitness value of the black hole (best solution);
- *f_i* is the fitness value of *i*th star;
- *N* is the total number of stars (solutions) in the population.

When the difference between a candidate solution's fitness value and the f_{BH} is less than *R*, the candidate solution is collapsed, and a new solution will be placed in the search space.

The pseudocode of BHA for university examination timetabling is shown in Fig. 2. Slight modification on the original BHA is needed to meet the university examination timetabling problem. This is done by modifying the movement of stars towards black holes using haploid crossover [37].

By referring to Fig. 2, the BHA for university examination timetabling problem is divided into two phases (initialization and improvement). In the first phase, the iteration for BHA is initialized (line 2), and the population is initialized based on the population size (lines 3-4). The solution initialization is done by using a combination of graph coloring heuristics proposed in [38]. Line 5 initializes the crossover point for haploid crossover. Lines 6-8 calculate the penalty value of each solution (*sol*_i) by using (1) and the best solution is assigned as black hole (*sol*_{bh}).

%Black Hole Algorithm%									
1. Initialization:									
2. Set number of iterations for BHA, iterBHA.									
Set the population size, N;									
 Initialize the population; 									
5. Set the crossover point, cp;									
Evaluates quality of each solution, f(sol);									
7. Select the solution with best fitness as black hole									
8. solbh, solbh - soli									
9.									
10.Improvement:									
11.For n = 1 to iteraBHA do									
12. For i = 1 to N do									
 //Step1: Move the star towards black hole 									
 Change the location of soli by moving towards 									
 black hole, sol_{bh} (haploid crossover). 									
16.									
17. //Step2: Update the location of black hole									
 If f(sol_i) < f(sol_{bh}) do 									
19. sol _{bh} ← sol _i									
20. End If									
21.									
22. //Step 3: Check if star crosses event									
 //horizon of the black hole 									
24. If $f(sol_i)-f(sol_{bh}) < R$ (calculate using (7)) do									
 soli collapsed and new solution (solnew) is 									
generated randomly;									
27. sol _i + sol _{new}									
28. End If									
29. End For									
30.End For									

Fig. 2 Pseudocode of BHA for university examination timetabling problem

There are three steps in the improvement phase (lines 11-30). The first step (lines 14-15) is the movement of the star (sol_i) towards the black hole (sol_{bh}) . Haploid crossover [37] is used in this step where the black hole is crossed over with the stars in the population. With this, the new solution generated will contain information of the black hole. A parameter is needed for the haploid crossover, i.e., crossover point, indicating the number of time slots involved in the crossover process. Besides, the feasibility of the timetable solution produced by haploid crossover must be preserved. Fig. 3 demonstrates the crossover process with a crossover point of 1 for the selected star (solution, sol_i) and the black hole (best solution, sol_{bh}).



Fig. 3 Haploid crossover process with crossover point 1

The shaded columns in Fig. 3 represent the timeslots selected for the haploid crossover process. To begin, timeslots t_4 and t_3 are selected from sol_i and sol_{bh} , respectively. Next, all moveable exams from timeslot t_3 of sol_{bh} (e_{21} , e_9 and e_{15}) are moved into timeslot t_4 of sol_i . Exam e_5 could not be moved because it clashes with event e_{25} (condition (a) violated). Lastly, a repair process is performed to ensure the new solution's feasibility (sol_i '). Here (refer to sol_i '), the exams e_9 and e_{21} are allocated into two different timeslots (t_4 and t_6 for e_9 ; t_1 and t_4 for e_{21}). Hence, the e_{22} scheduled on t_1 , and the e_9 scheduled on t_6 are removed to avoid duplication.

In step 2 (lines 18-20), the fitness of the star (sol_i) after movement in step 1 is compared against the black hole (sol_{bh}). A star will be selected as a new black hole if its penalty value is better (lower for the problem studied) than the current black hole. Lastly, in step 3 (lines 24-28), a star is considered crossed the event horizon of the black hole if the distance (difference in terms of penalty value) between the black hole and a star is smaller than R (calculated using (7)), the star (sol_i) will collapse, and a new random solution (solnew) will be placed in the search space. Hence, it can be concluded that the exploitation ability of BHA is controlled by the movement of the star towards the black hole since information of black hole is incorporated into the star, resulting in the search exploring toward promising search regions for better quality of solution. As for the exploration ability, it is done by generating and placing a new star in the search space if a star crosses the event horizon of the black hole. This is to escape local optima and explore towards unvisited search regions.

Despite BHA possessing exploitation and exploration abilities, there is still inefficiency in terms of exploitation where the movement of the star towards black hole is relatively slow, and the replacement of the star that crossed the horizon has led to slow convergence of the solution searching process. Hence, the exploitation of BHA needs to be strengthened.

C. Hill Climbing (HC)

Hill climbing local search [39] is the simplest local search approach that iteratively examines neighborhood solutions of the current solution and accepts candidate solutions using a greedy selection scheme. The stopping criteria for HC is usually based on a predefined number of iterations. Hence, HC is an approach that possesses a strong exploitation ability in exploring the search region of an individual solution. Other than that, HC's performance greatly depends on the use of neighborhood generation mechanisms in exploring neighborhood solutions of a particular search region [40]. The pseudocode of HC is presented in Fig. 4.

Fig. 4 Pseudocode for HC local search

The neighborhood generation mechanism used by HC to produce tentative solutions for university examination timetabling problems is as follows.

NM1: select an exam e_1 and a time slot *t* randomly (*t* cannot be the same as the existing time slot for e_1), move e_1 to *t*

NM2: select two different exams (e₁ and e₂) randomly and exchange their time slots

NM3: select two different exams (e_1 and e_2) and two-time slots randomly (t_1 and t_2), move e_1 to t_1 and e_2 to t_2

NM4: two-time select slots $(t_1 \text{ and } t_2)$ randomly, exchange their exams

NM5: select an exam e_l randomly and move it to a different time slot *t*. If there is any conflicting exam to e_l in time slot *t*, move the conflicting exam to the time slot of e_l (t_{el}). In turn, all exams (in t_{el}) that in conflict with them will be shifted to time slot *t*. This process will repeat until conflict-free is achieved.

The feasibility of the solution is always maintained during the neighborhood solution generation, and this means that the solution searching is limited to the search space with feasible solutions.

D. The Proposed Algorithm

In this section, the proposed hybrid algorithm is discussed. The BHA is hybridized with HC (called hybrid BHA) to improve the exploitation ability of BHA in searching for local optimal solutions. Experiment as shown in Fig. 5, demonstrates how the HC is incorporated into BHA. In the hybrid BHA, the HC (line 18) is invoked to improve a solution after the haploid crossover process in Step 1 (lines 14-15).

```
%Hybrid Black Hole Algorithm%

    Initialization:

  Set number of iterations for BHA, iterBHA.
  Set the population size, N;
4. Initialize the population;
5. Set the crossover point, cp.
6. Evaluates quality of each solution, f(sol);
7. Select the solution with best fitness as black hole
8. solbh, solbh + soli
10. Improvement:
11. For n = 1 to iteraBHA do
   For i = 1 to N do
12.
13
       //Step1: Move the star towards black hole
       Change the location of sol: by moving towards
14
15
      black hole, sol<sub>bh</sub> (haploid crossover).
16
17.
       //Step2: Hill Climbing local search
18
       sol₁ ← HC (sol₁)
19
20
       //Step3: Update the location of black hole
21
      If f(soli) < f(solin) do
22
        sol<sub>bh</sub> ← soli
23
      End If
24
25.
       //Step4: Check if star crosses event horizon
26.
       //of the black hole
27
       If f(sol_1)-f(sol_{bh}) < R (calculate using (7)) do
28
        sol1 collapsed and new solution (solnew) is
29.
        generated randomly;
30.
        sol₁ ← solnem
31.
      End If
32. End For
33.End For
```

Fig. 5 Pseudocode for hybrid BHA

The HC will try to enhance the quality of working solution by examining neighborhood solutions of working solution iteratively until a local optimum is obtained. With this, the exploitation ability of BHA is enhanced as the solution searching around the black hole (current best solution) is strengthen. Next, the search will continue with black hole update process in step 3 (lines 21-23) and crossing of event horizon checking in step 4 (lines 27-31). It will then repeat from Step 1 to Step 4 until the maximum iteration used for BHA is reached.

III. RESULTS AND DISCUSSION

The university examination timetabling problem presented in Section II is used to examine the performance of hybrid BHA. Besides, the hybrid BHA is programmed using python under Windows 10 with Core i7 3.20 GHz of processor and 16 GB of RAM. Table IV presents the parameters used in the BHA and hybrid BHA which are established after some preliminary experiments.

TABLE IV PARAMETER SETTINGS FOR BHA AND HYBRID BHA

Parameters	BHA	Hybrid BHA
Population size	10	10
Number of iteration	1,000	1,000
HC local search iteration	-	2,000
Haploid crossover, cp	-	8

The parameter settings for BHA and hybrid BHA are presented in Table IV. The experiments are performed with a population size of 10, number of iterations of 1000, crossover point (cp) of 8 are used for both BHA and hybrid BHA. As for HC, the number of iterations used is 2000 for hybrid BHA. Preliminary experiments on the number of iterations for HC and cp have been conducted. From the preliminary result on the number of iterations used for HC, it is found that the degree of exploitation of HC is depending on the number of iterations. A higher value for number of iterations will result in a stronger exploitation power. However, the improvement on the performance is not significant when the number of iterations above 2000 is used.

Besides, it is observed that the value of crossover point (cp) plays a crucial role in influencing the search characteristics of BHA and hybrid BHA. A low cp will result in a higher degree of random exploration as the information from the black hole to a candidate solution is lesser, resulting in the search exploring the search region blindly and slow convergence occurred. In contrast, a higher cp will result in a higher degree of exploitation as more information from the black hole is shared to the candidate solution and the search will focus intensively on the search region closer to the black hole, which then reduces the diversity of the population and results in premature convergence. Besides, reduced diversity may also lead the search cyclically exploring same search region.

A cp of 8 is used as it gives the best performance in maintaining the balance of exploration and exploitation.

A. Results Comparison Between BHA and Hybrid BHA

Experimental results for 30 runs on both BHA and hybrid BHA and the percentage of improvement achieved by hybrid BHA over BHA are illustrated in Table V and the best recorded penalty values are shown in bold. Based on Table V, it shows that hybrid BHA outperforms BHA in all instances.

TABLE V	
COMPARISON OF THE RESULTS BETWEEN BHA AND HYBRID BHA	4

Detesets	BH	[A	Hybrid	I BHA	Improvement
Datasets	Best	Avg.	Best	Avg.	(%)
car92	7.65	7.75	3.81	3.84	67.02
car91	6.43	6.53	4.51	4.53	35.10
ear83	46.55	46.81	32.10	32.55	36.75
hec92	14.48	14.63	9.56	10.00	40.93
kfu93	20.01	20.11	12.85	13.00	43.58
lse91	17.20	17.29	9.73	10.02	57.39
sta83	159.01	159.65	157.03	157.07	1.25
tre92	11.15	11.20	7.78	7.84	35.60
uta92	5.03	5.09	3.10	3.13	47.48
ute92	31.23	31.34	24.50	24.84	24.15
yor83	45.67	46.10	34.86	35.01	26.85

Fig. 6 demonstrates the convergence graph for BHA and hybrid BHA for instance *hec92*. The *x*-axis represents the number of iterations and the *y*-axis represents the penalty cost value.



From Fig. 6, there is a steep curve slope for hybrid BHA compared to BHA, indicating great improvement at the beginning of the search. Besides, the graph shows that the convergence speed of hybrid BHA is better than BHA and achieves a better quality of solutions across the search process than BHA.

 TABLE VI

 RESULTS COMPARISON WITH HYBRID APPROACHES

Dataset	Hybrid	H1	H2	H3	H4	Н5	H6	H7	H8	Н9	H10	H11	H12	H13	H14
	BHA	[25]	[27]	[31]	[22]	[14]	[13]	[23]	[32]	[7]	[21]	[24]	[19]	[36]	[26]
car92	3.81	4.59	4.60	4.77	3.90	4.10	4.11	4.54	4.16	4.32	4.39	3.88	3.89	4.22	3.68
car91	4.51	5.41	5.66	5.30	4.60	4.80	4.81	5.38	5.16	5.11	5.33	4.38	4.79	5.00	4.31
ear83	32.10	33.64	37.00	38.39	32.80	34.92	36.10	36.27	35.86	33.56	35.17	33.34	33.43	34.07	32.48
hec92	9.56	10.15	11.96	12.01	10.00	10.73	10.95	10.73	11.94	11.62	11.19	10.39	10.49	10.36	10.03
kfu93	12.85	13.49	15.44	15.09	13.00	13.00	13.21	15.17	14.79	15.81	14.07	13.23	13.72	14.01	12.81
lse91	9.53	10.44	10.82	12.72	10.00	10.01	10.20	11.87	11.15	11.32	11.89	10.52	10.29	11.01	9.78
sta83	157.03	157.03	157.77	158.10	156.90	158.26	159.74	158.16	159.00	158.88	157.39	157.06	157.07	157.04	157.03
tre92	7.78	8.34	8.18	8.74	7.90	7.88	8.00	8.86	8.6	8.52	9.41	7.89	7.86	8.38	7.66
uta92	3.10	3.72	3.88	3.32	3.20	3.20	3.32	3.59	3.59	3.21	3.89	3.13	3.10	3.40	3.01
ute92	24.50	24.81	28.78	30.32	24.80	27.00	26.17	35.34	28.30	28.00	27.11	25.12	25.33	25.80	24.82
yor83	34.86	36.65	40.51	40.24	34.90	36.22	36.23	40.26	41.81	40.71	40.76	35.49	36.12	36.95	34.45

 TABLE VII

 Results comparison with meta-heuristic approaches

Dataset	Hybrid	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12	M13
	BHA	[41]	[42]	[43]	[1]	[44]	[45]	[46]	[9]	[47]	[48]	[49]	[50]	[11]
car92	3.81	3.93	5.90	3.67	3.82	4.40	4.10	4.20	4.40	4.30	4.22	4.29	4.22	3.64
car91	4.51	4.50	-	4.32	4.58	5.40	4.65	4.80	5.20	5.20	4.92	4.99	5.14	4.24
ear83	32.10	33.71	-	32.62	33.23	34.80	37.05	35.40	34.90	36.80	35.87	34.42	34.38	32.42
hec92	9.56	10.83	11.54	10.06	10.32	10.80	11.54	10.80	10.30	11.10	11.15	10.40	10.49	10.03
kfu93	12.85	13.82	-	12.80	13.34	14.10	13.90	13.70	13.50	14.50	14.37	13.50	13.64	12.80
lse91	9.73	10.35	-	9.78	10.24	14.70	10.82	10.40	10.20	11.30	10.89	10.48	10.92	9.77
sta83	157.03	151.52	151.16	157.03	157.14	-	168.73	159.10	159.20	157.30	157.81	157.04	157.07	157.03
tre92	7.78	7.92	7.97	7.64	7.74	8.70	8.35	8.30	8.40	8.60	8.38	8.16	8.52	7.59
uta92	3.10	3.14	3.57	2.98	3.13	-	3.20	3.40	3.60	3.5	3.35	3.43	3.47	2.95
ute92	24.50	25.39	24.08	24.78	25.28	25.40	25.83	25.70	26.00	26.40	27.24	25.09	34.38	24.76
yor83	34.86	36.53	-	34.71	35.68	37.40	37.28	36.70	36.20	39.40	39.33	36.86	37.95	34.40

Again, this demonstrates that the hybridization of HC after the solutions (stars) explore towards the best solution (black hole) has significantly improved the exploitation ability as well as the convergence behavior of BHA. Besides, it is also believed that using multiple neighborhood structures has improved HC local search's ability to explore the neighborhood solutions.

B. Results Comparison with Hybrid Approaches

In this section, a comparison between hybrid BHA with other hybrid approaches is carried out. In the last decade, timetabling researchers have proposed many hybrid approaches. In this study, hybridization between BHA and HC, called hybrid BHA, is proposed to improve the exploitation ability of BHA. Table VI presents the comparison with previous hybrid approaches that were applied using the same dataset, with the best results presented in bold. From Table VI, the proposed hybrid BHA obtained the best results for ear83, ute92, lse91 and hec92 instances, achieving the second rank for car92, kfu93, tre92, and sta83, respectively *uta92* and *yor83* instances, and the third rank for *car91* instance.

C. Results Comparison with Meta-heuristic Approaches

Various meta-heuristic approaches have been applied in solving Carter's dataset in recent years. Hence, it is important to have the comparison between the proposed hybrid BHA with the meta-heuristic approaches tested on the same dataset (refer to Table VII). The results indicated in bold are the best and the "-" shows that the instance is not tested.

Table VII shows that the proposed hybrid BHA performed better than other meta-heuristic approaches in three instances, which are *ear83*, *hec92* and *lse91*. Besides, hybrid BHA ranked second for *ute92*, ranked third for *car92*, *car91*, *kfu93*, *sta83*, *tre92*, *uta92* and *yor83* instances.

D. Comparison with the Best-known Results

In this section, a comparison between the hybrid BHA and best-known results reported for the university examination timetabling dataset is presented in Table VIII. Literature-beating results have been produced for three instances (*ear83*, *hec92* and *lse91*).

 TABLE VIII

 COMPARISON OF THE RESULTS BETWEEN HYBRID BHA WITH BEST-KNOWN

Datasets	Hybrid	BHA	Duariana Daat Vnamn		
	Best	Avg.	r revious desi	Known	
car92	3.81	3.84	3.64	[11]	
car91	4.51	4.53	4.24	[11]	
ear83	32.10	32.55	32.42	[11]	

ð	30.70	30.20	39.40	39.33	30.80	37.95	34.40
	hec92	9.	56	10.00	10.03	[1	1], [26]
	kfu93	12.	.85	13.00	12.80	[1]	1], [43]
	lse91	9.	73	10.02	9.77		[11]
	sta83	157.	.03	157.07	151.16		[42]
	tre92	7.	78	7.84	7.59		[11]
	uta92	3.	10	3.13	2.95		[11]
	ute92	24.	50	24.84	24.08		[42]
	yor83	34.	.86	35.01	34.40		[11]

Besides, the overall performance of hybrid BHA is also good as it achieved the second rank when compared with 14 hybrid approaches and third rank when compared with 13 meta-heuristic approaches as presented in Tables IX and X, respectively.

 TABLE IX

 Average ranking (Friedman Test) of the comparison with hybrid approaches on university examination timetabling problem

No.	Approaches	Mean Rank	Ranking
1	Hybrid BHA	1.37	2
2	H1	5.87	7
3	H2	9.07	14
4	H3	9.23	15
5	H4	2.53	3
6	H5	4.90	6
7	H6	6.10	9
8	H7	9.00	13
9	H8	8.57	11
10	H9	7.93	10
11	H10	8.67	12
12	H11	3.53	4
13	H12	3.90	5
14	H13	6.00	8
15	H14	1.33	1

TABLE X AVERAGE RANKING (FRIEDMAN TEST) OF THE COMPARISON WITH META-HEURISTIC APPROACHES ON UNIVERSITY EXAMINATION TIMETABLING PROBLEM

No.	Approaches	Mean Rank	Ranking
1	Hybrid BHA	2.07	3
2	M1	4.50	6
3	M2	3.39	4
4	M3	2.04	2
5	M4	3.71	5
6	M5	7.00	9
7	M6	7.46	12
8	M7	6.46	8
9	M8	7.04	10
10	M9	9.18	14
11	M10	8.11	13
12	M11	5.75	7
13	M12	7.39	11
14	M13	1.39	1

Friedman statistical test (with a significance level of 95%, $\alpha = 0.05$) has also performed to show the significant differences between hybrid BHA and the approaches in comparison reported in Sections V.B and V.C. The computed *p*-value are 4.20×10^{-18} and 8.53×10^{-15} for the results presented in Tables IX and X, respectively. This proves that there is a significant difference in terms of performance between the hybrid BHA and the other approaches under comparison.

IV. CONCLUSION

This paper proposes a hybrid approach, hybrid BHA, to improve the basic BHA for the university examination timetabling problem. Besides, this study has also demonstrated how BHA can be applied as an alternative approach to solve the problem. Despite BHA possess good exploration ability, it is still ineffective for the university examination timetabling problem as exploitation ability is poor. It should be noted that exploitation is an important search ability in exploring the search regions of the population. Therefore, hybrid BHA is introduced to enhance the exploitation by integrating HC in the basic BHA.

In the HC local search, several neighborhood generation mechanisms were used. This is to improve the exploration power of HC in examining neighborhood solutions of a working solution. The performance of hybrid BHA is examined using Carter's examination benchmark dataset. Experimental results show that the hybrid BHA is capable of producing competitive results for all the problem instances. A comprehensive analysis against 27 approaches published in the literature has been conducted. Findings from the Friedman test show that the hybrid BHA ranked second and third in comparison with hybrid and meta-heuristic approaches. In addition, the Friedman test also proves that the performance of hybrid BHA is significantly different from the approaches under comparison. Importantly, the hybrid BHA produced three best results compared to the best-known reported.

Lastly, future work can study on integrating other exploitation mechanisms into hybrid BHA. Besides that, the application, and the effectiveness of using different neighborhood generation mechanisms used in HC can also be studied.

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